Deconstructing (Future) Quantum Computer Design & Use

(Where does the power of Quantum Computing stem from? How will it impact Security/Privacy)

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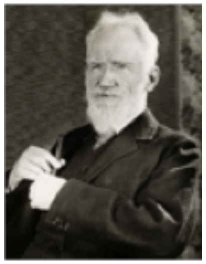
A Big *Gedanken Experiment*

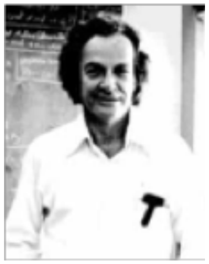
- Large-scale quantum computers
  - Do not exist but not known to violate underlying physics
  - Justified by entirely new (fascinating) form of computation, big challenges

- George Bernard Shaw, 1938
  - *"You have nothing to do but mention the quantum theory and people will take your voice for the voice of science and believe anything"*



- Scott Aaronson, MIT computer scientist, 2011
  - *"Quantum Mechanics, contrary to its reputation, is actually really simple, once you take all the Physics out."*

- Richard Feinman, 1965
  - *"I think I can safely say that nobody understands quantum mechanics."*
  - **What else did he say?**



Outline (Sharp Left Turn)

- **Qubits, Quantum State**
- **Quantum Circuits**

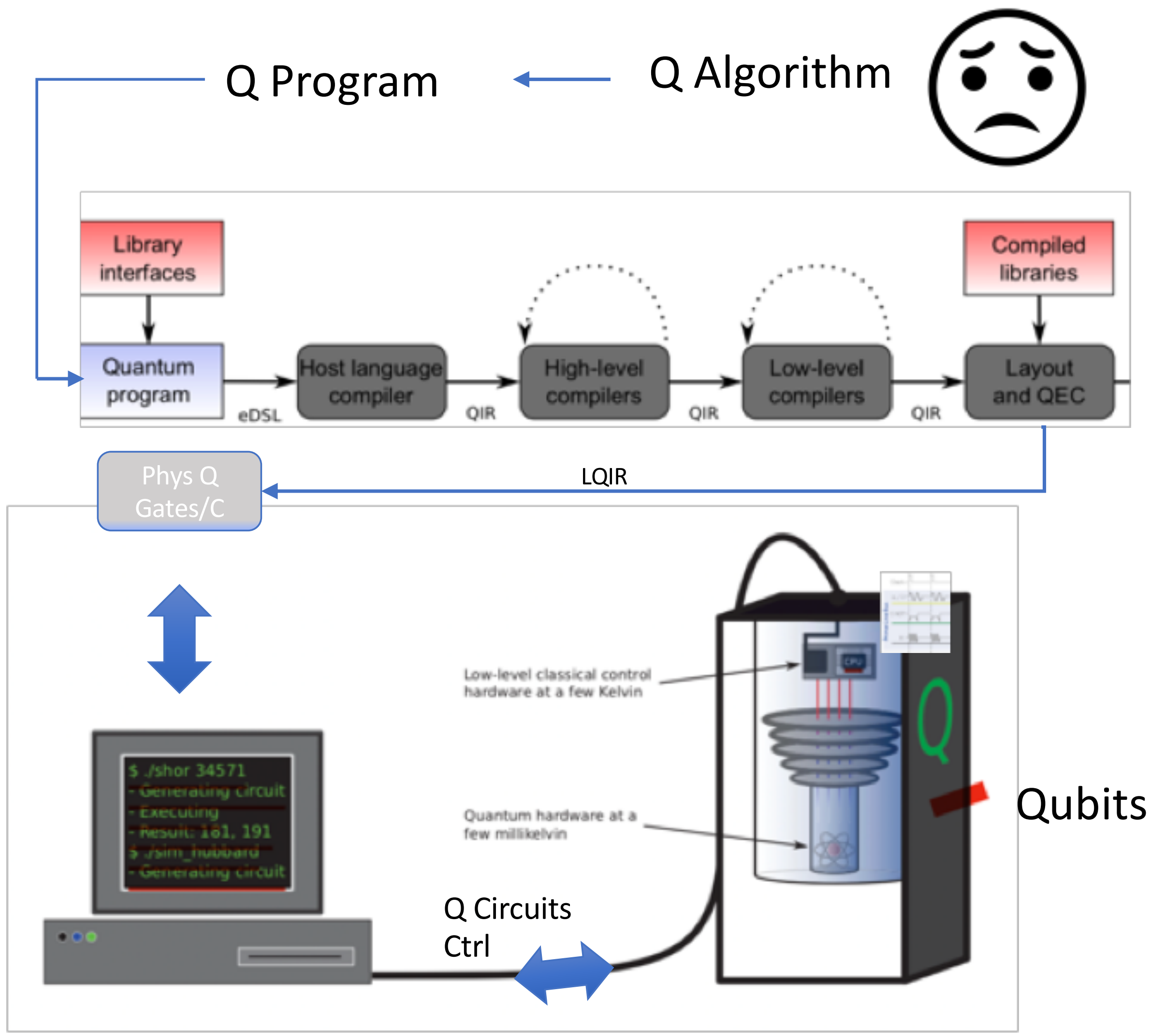
} *Foundations*

- Q Algorithms
- Q Compilers (QEC, Circuit Synth)
- Runtime Models
- C Microarchitecture Ctrl

Thomas Barbey photography

Viewpoint: (Future) Quantum Computer Design (Much) More Than Quantum Hardware Design

- Runs a quantum algorithm on quantum hardware
- Controlled by classical computer - feeds it with things to do (Q circuits)
- Q circuits - Q compiler generated – transforms state of special kind of bits called qubits



Part of Figure from (Haner, 2016)

Qubits, Quantum Information

- A classical bit can be 0 or 1.
- A qubit is a two-level quantum system. Its *quantum state is* a superposition of 0 and 1 states.
  - $N = 1$: $|\Psi\rangle = c_0|0\rangle + c_1|1\rangle$ Dirac "ket" notation
  - $N = 2$: $|\Psi\rangle = c_0|00\rangle + c_1|01\rangle + c_2|10\rangle + c_3|11\rangle$
    - c_i - complex coefficients called amplitudes; $|00\rangle$ etc., standard *basis states*

$$|\Psi\rangle = \sum_{i=0}^{2^n-1} c_i \left| b_{i,n-1} b_{i,n-2} \ldots b_{i,0} \right\rangle \quad (1)$$

- Why noteworthy?
  - N qubits can "store" exponentially more info than classical N bits due to superposition => enables *Quantum parallelism*
  - Wave-like properties => non-classical operations like *entanglement, interference*

Superposition: Imagine You Cannot Make Up Your Mind on Deserts, *State.* Which Are You *Inclined* to Get?

Qubits, Quantum Information contd.

- How to implement a qubit?
  - Clockwise and counterclockwise current in a superconducting circuit
  - Up and down electron spin in a uniform magnetic field
  - Horizontal and vertical polarization of a photon
  - Ground and excited state of a trapped ion

- Issues
  - Subject to quantum noise or decoherence => fragile
  - Output is *only* classical – so called *measurements* necessary to read out

Vector Representation of State

- State of a qubit can also be represented as a two dimensional complex vector space

$$\alpha|0\rangle + \beta|1\rangle = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\end{pmatrix}$$

  - where $|0\rangle$ and $|1\rangle$ are the basis states of a two-dimensional complex vector space and α, $\beta \in C$
- For two qubits, the basis states are all possible configurations of two classical bits, i.e. $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$. A general two-qubit state:

$$\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$
$$= \alpha\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\\0\\0\end{pmatrix} + \gamma\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + \delta\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \begin{pmatrix}\alpha\\\beta\\\gamma\\\delta\end{pmatrix}$$

State of N-qubit System

- The state of a general n-qubit system can be an *arbitrary* superposition over all 2^n computational basis states, i.e.,

Dirac's *vector*

- $$\sum_{q_1 q_2 \ldots q_n \in \{0,1\}^n} c_{q_1 \ldots q_n} |q_1 \ldots q_n\rangle = \sum_{i=0}^{2^n-1} c_i |i\rangle \qquad \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ \ldots \\ c_{2n\_1} \end{pmatrix}$$

- Complex amplitudes c_i need to satisfy the normalization condition:

$$\sum_i |c_i|^2 = 1$$

- Probability of measuring a given state is equal to lulus of the amplitude associated with that state. **Bohr's rule**.

Joint Quantum State from Individual States

- Example two qubits:
  - $|\alpha\rangle := \alpha_0|0\rangle+\alpha_1|1\rangle$ and $|\beta\rangle := \beta_0|0\rangle+\beta_1|1\rangle$,
  - the state of the entire system is the tensor product of the two individual states
    - which corresponds to the Kronecker product in vector notation

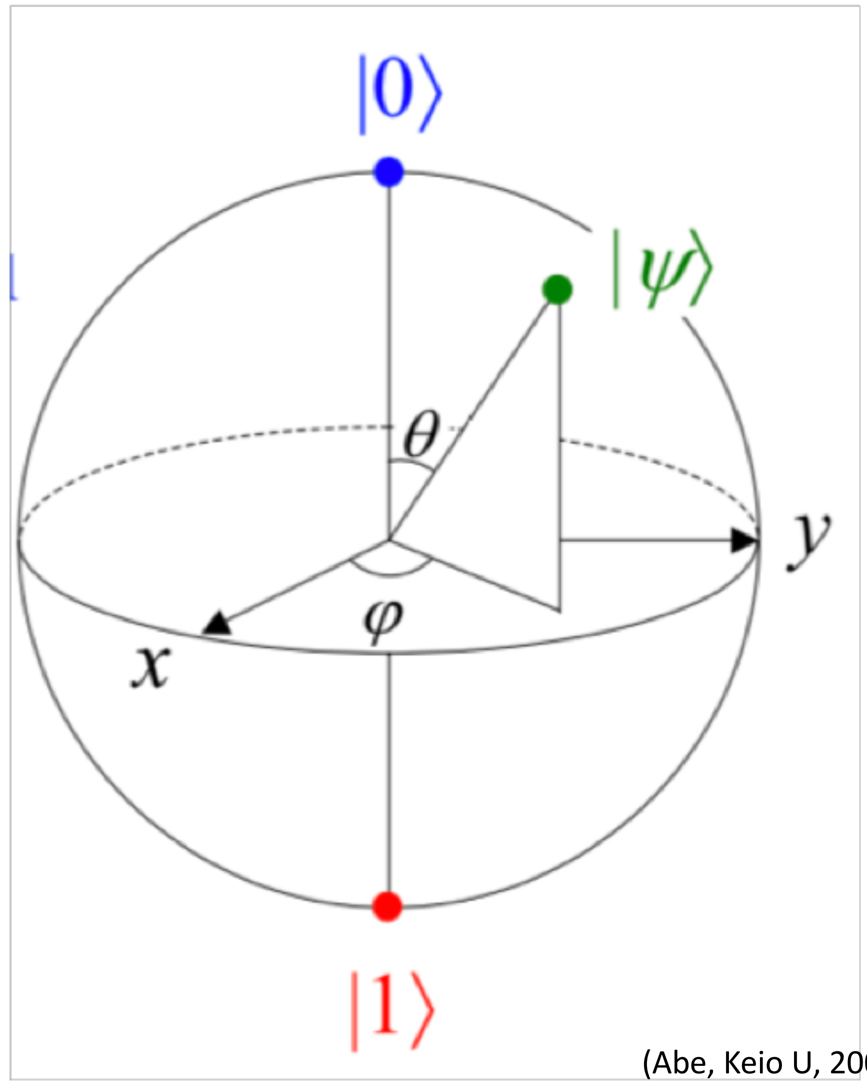
$$\begin{pmatrix}\alpha_0\\ \alpha_1\end{pmatrix} \otimes \begin{pmatrix}\beta_0\\ \beta_1\end{pmatrix} = \begin{pmatrix}\alpha_0\beta_0\\ \alpha_0\beta_1\\ \alpha_1\beta_0\\ \alpha_1\beta_1\end{pmatrix}$$

  - Or,
    - $= \alpha_0\beta_0\ |00\rangle + \alpha_0\beta_1\ |01\rangle + \alpha_1\beta_0\ |10\rangle + \alpha_1\beta_1\ |11\rangle$ *in Dirac's*

Block Sphere Representation of a Qubit

- Intuitive to understand quantum transformations
- $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$

- The state is visualized as a point/vector on the sphere.
- A gate is a transformation to another point/vector.



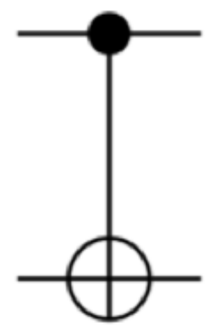
(Abe, Keio U, 2005)

Qubit Gates

- A gate is a unitary transformation (thus reversible) on the quantum state
  - $|\psi\rangle \rightarrow U|\psi\rangle = |\psi'\rangle$
  - U is the unitary operator or matrix

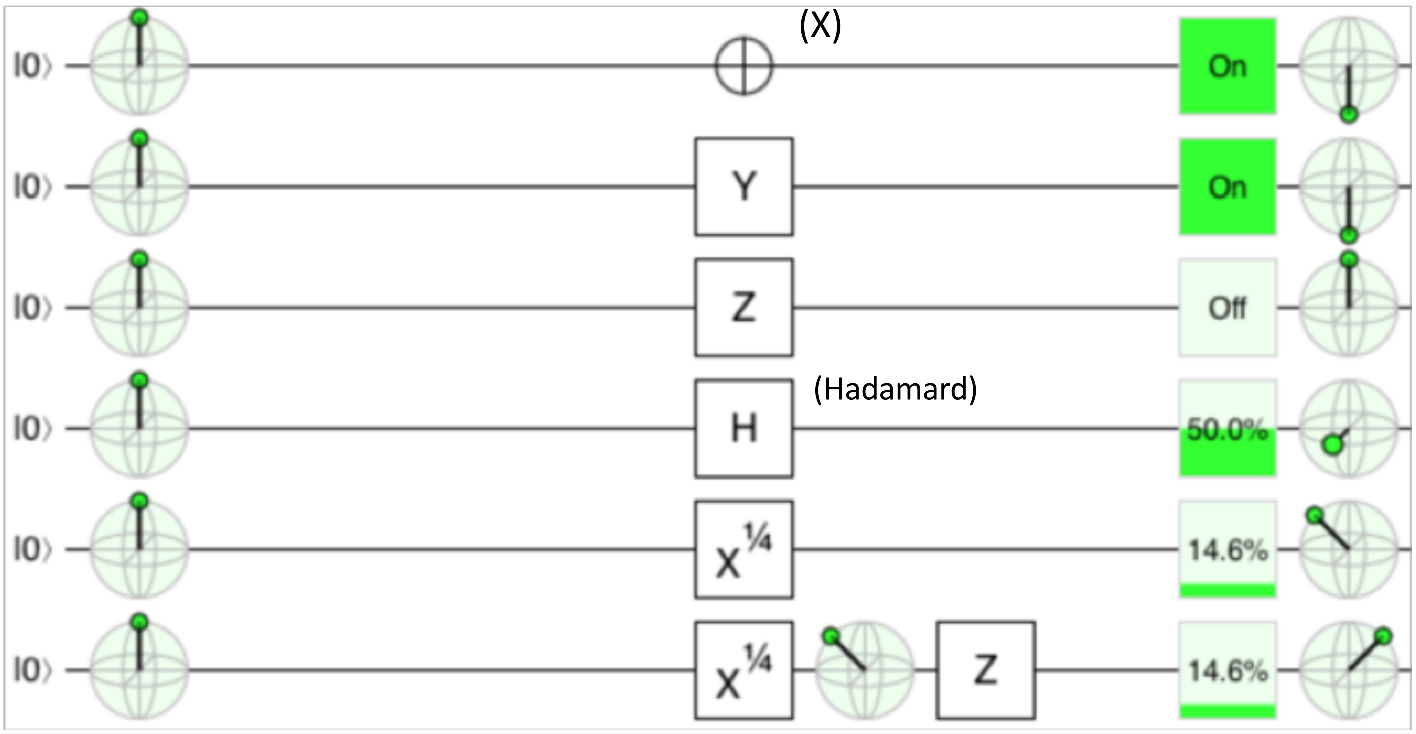
- A unitary quantum operation on n qubits can be written as a matrix of dimension $2^n \times 2^n$.

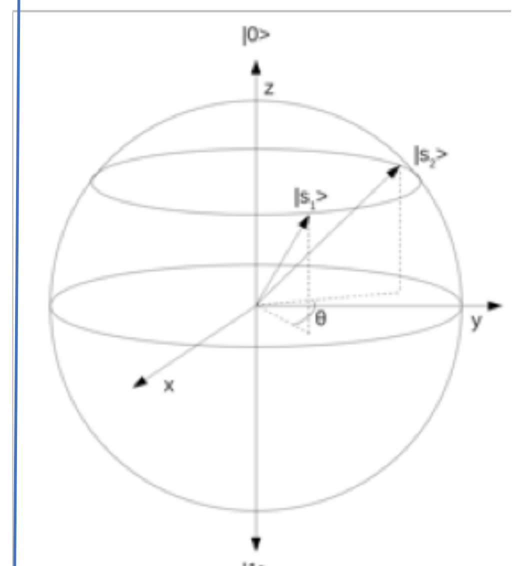
Some Important Q Gates

Gate	Matrix	Symbol
NOT or X gate	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	
Y gate	$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	Y
Z gate	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Z
Hadamard gate	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	H
S gate	$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$	S
T gate	$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$	T
Rotation-Z gate	$\begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$	Rz_θ
Controlled NOT (CNOT)	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	

Gates with *Quirk* Circuit Simulator

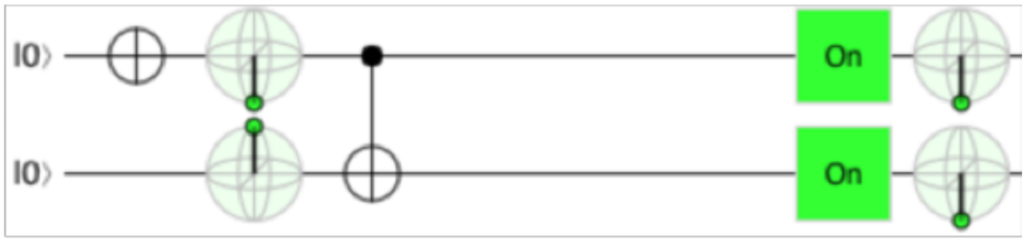
- Created to show effect with Bloch



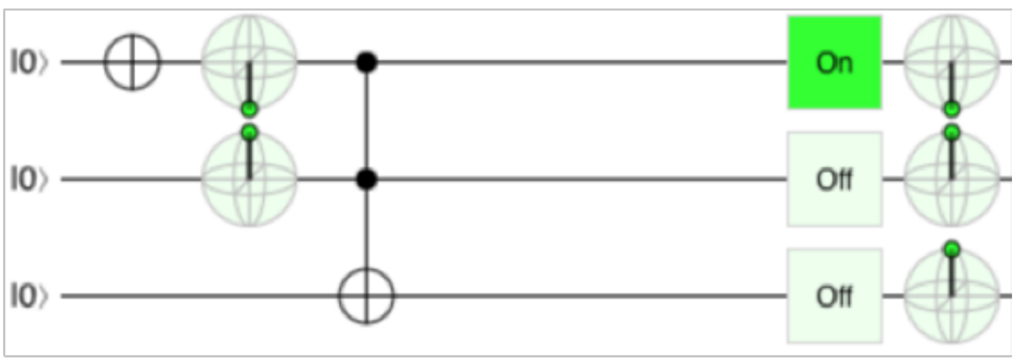


Phase Shift
(Z, S, T)

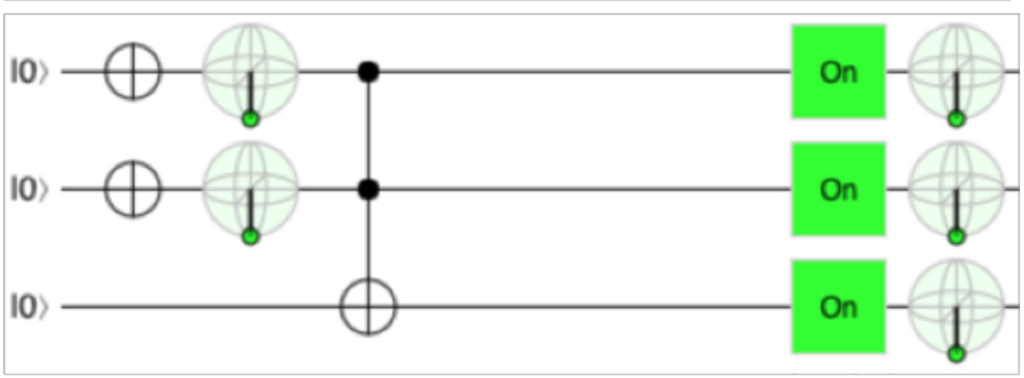
Two-input CNOT



3-input CCNOT or Toffoli

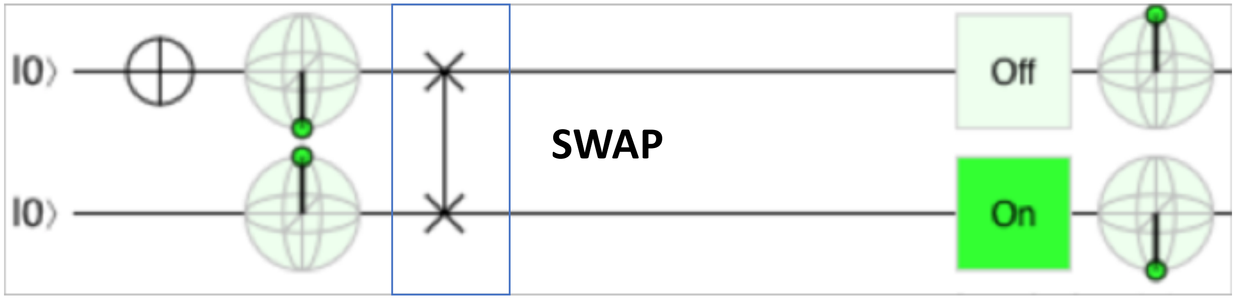


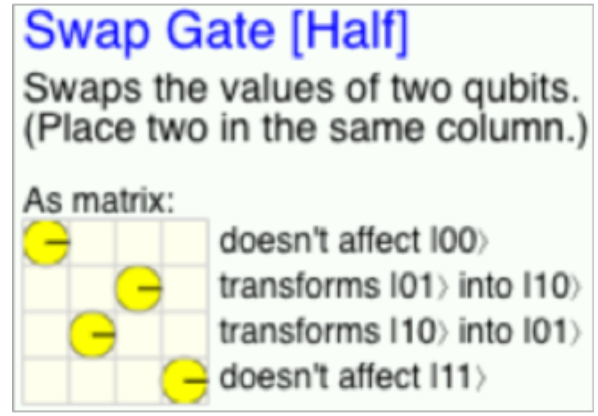
Nothing happens on 3rd

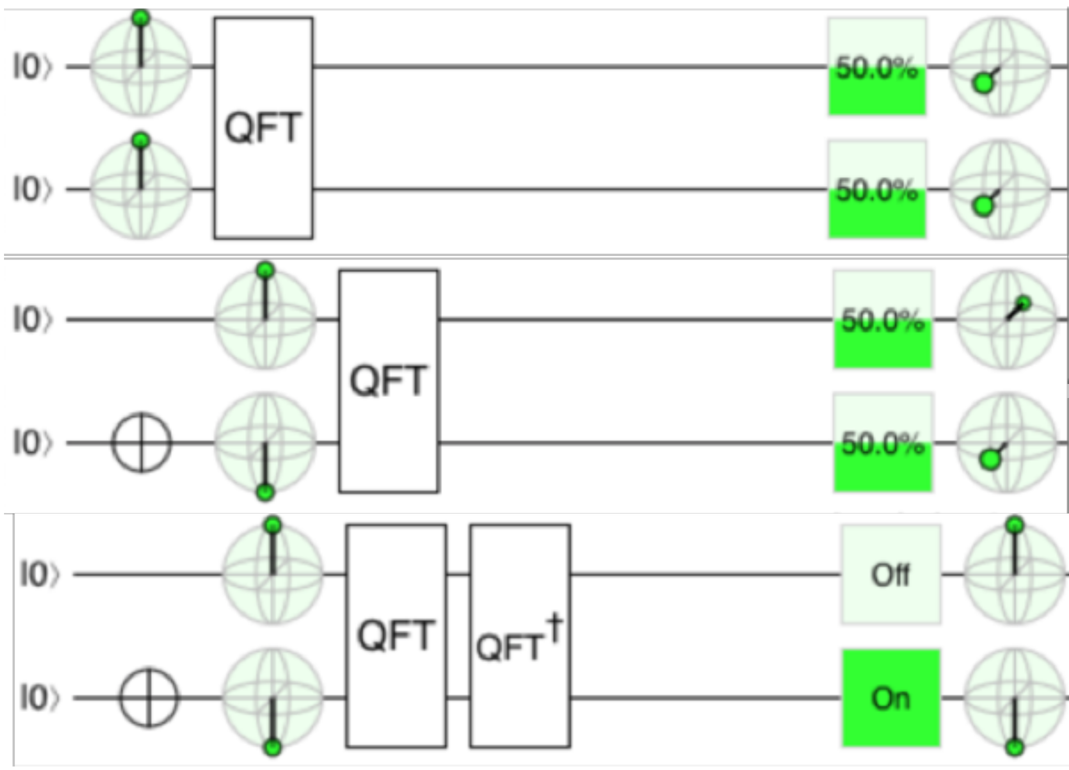


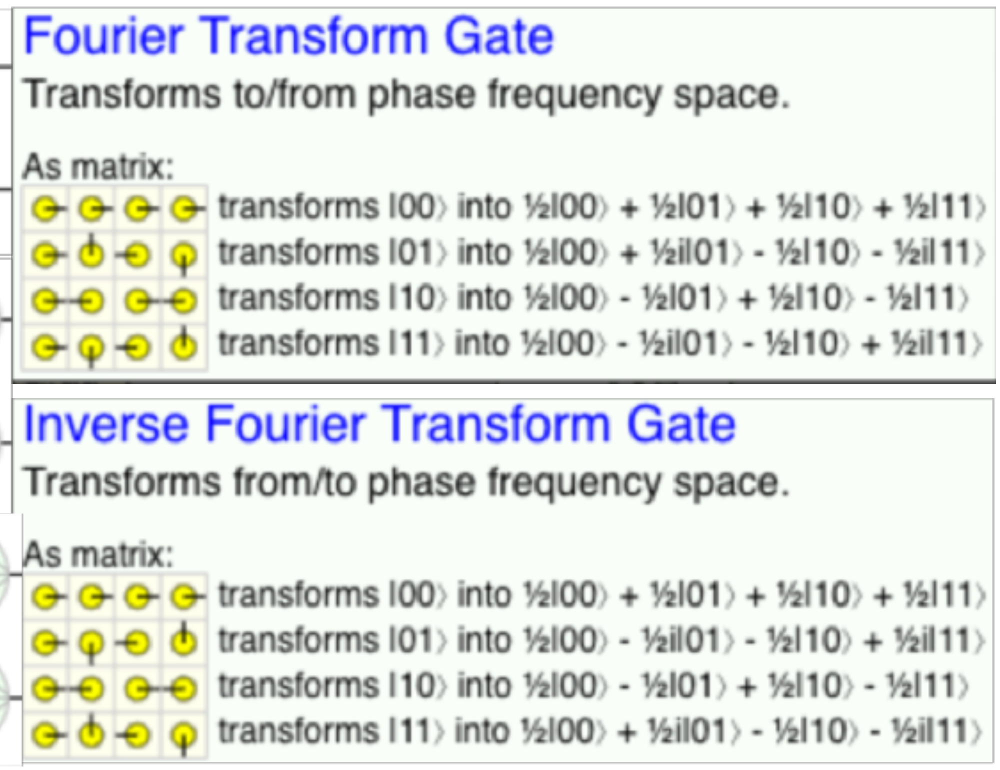
Inverts on 3rd qubit

SWAP, QFT/QFT-1, Reversibility









What is Needed in a Q Computer? Quantum Universality Q Gate Sets

- All you ever need for a quantum computer - could approximate any other quantum gate (unitary operation) at any precision
  - Toffoli and Hadamard already constitute a quantum *universal set*
- Other proven Q universal sets exist:
  - Hadamard, *S*, *T*, and CNOT; Toffoli, Hadamard, and π/4 –gate; CNOT, Hadamard, and π/8 –gate; Three-qubit Deutsch gate,…

- Solovay-Kitaev Theorem, any universal set of gates can simulate any other with *at most a polynomial increase in the # of gates*
  - If we're doing complexity theory, it really doesn't matter which universal gate set.
  - If we are designing a quantum computer it makes a lot of difference!

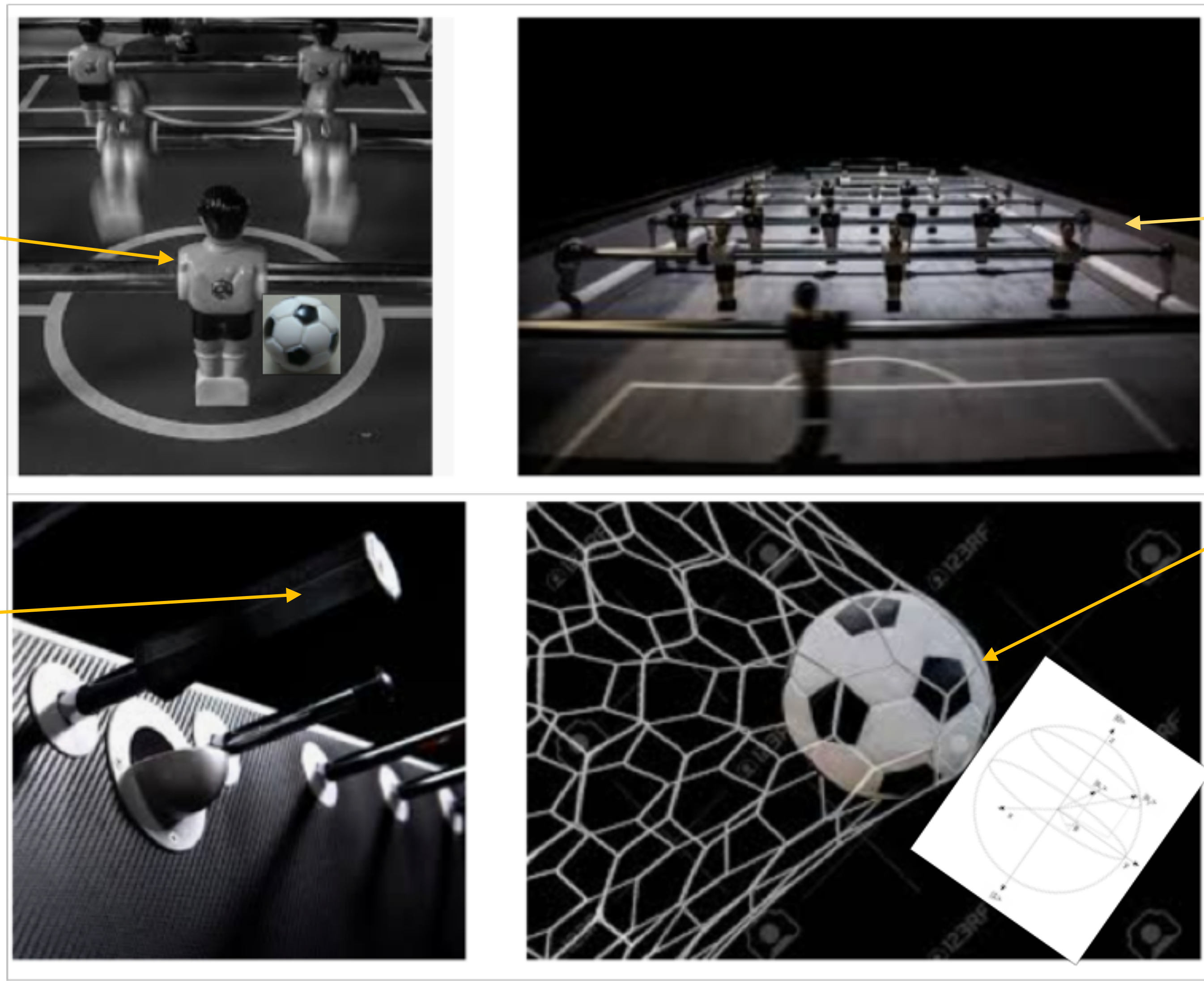
Quantum Circuits

- Sequence of quantum gates, each performing a unitary transformation on the Q state of registers

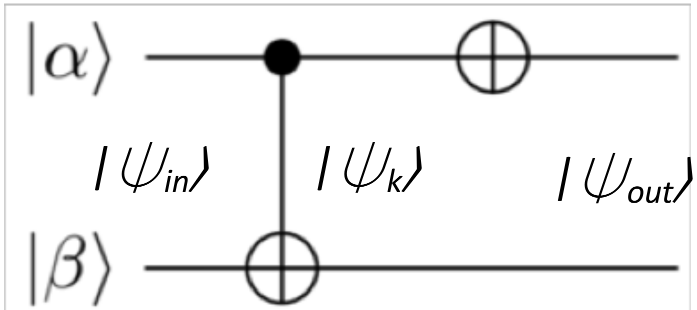
- Same number of inputs as outputs
- No loops! No cloning! But control flow...

- Read left to right; wires are qubits and symbols on each wire are gates; gates can act on one or more qubits
  - Hard to implement many-qubit gates …so often everything is built up from smaller gates (just like classical). E.g., n-QFT $O(n^2)$ in H, Ctrl phase shift gates
  - Connectivity – not all qubits may be used in CNOT
  - QEC – use many qubits as one logical one

Game of Table Soccer (w/ Block Sphere)?



Let's Analyze a 1st Simple Circuit:
(CNOT on joint two-qubit, then NOT on top)



Q Circuit

$|\psi_{in}\rangle = \alpha_0\beta_0\,|00\rangle + \alpha_0\beta_1\,|01\rangle + \alpha_1\beta_0\,|10\rangle + \alpha_1\beta_1\,|11\rangle$

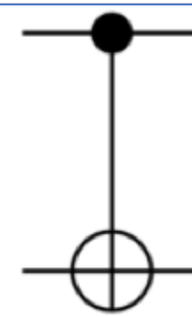
- Applying CNOT: matrix-vector multiplication using the unitary matrix of CNOT and joint input state vector

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\cdot\begin{pmatrix}\alpha_0\beta_0\\\alpha_0\beta_1\\\alpha_1\beta_0\\\alpha_1\beta_1\end{pmatrix}=\begin{pmatrix}\alpha_0\beta_0\\\alpha_0\beta_1\\\alpha_1\beta_1\\\alpha_1\beta_0\end{pmatrix}$$

$|\psi_{in}\rangle$ $|\psi_{k}\rangle$

Controlled NOT (CNOT)

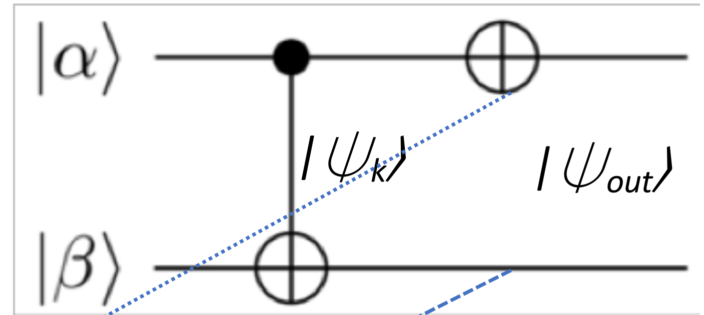
$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$



- Next, apply NOT on first qubit only

NOT or X gate	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\oplus$

Q Circuit Example Contd.



- NOT or X gate is applied on top qubit, nothing on bottom

$$X \otimes \mathbb{1}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad (1)$$

$$|\psi_{out}\rangle = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \alpha_0\beta_0 \\ \alpha_0\beta_1 \\ \alpha_1\beta_1 \\ \alpha_1\beta_0 \end{pmatrix} = \begin{pmatrix} \alpha_1\beta_1 \\ \alpha_1\beta_0 \\ \alpha_0\beta_0 \\ \alpha_0\beta_1 \end{pmatrix} \quad (2)$$

$|\psi_k\rangle$ $|\psi_{out}\rangle$

Quantum Parallelism - Intuition

- As a consequence of the linearity of quantum mechanics, gates are simultaneously applied to all basis states of a superposition at once
  - Classical SIMD operations, e.g., multimedia instruction set (ISA)
- If, for example, n qubits are in a complete superposition over all 2^n basis states, all possible outputs of a function f(x) can be calculated using only one function call:
- $$f\left(\sum_{i=0}^{2^n-1} c_i \left|x\right\rangle\right) = \sum_{i=0}^{2^n-1} c_i \left|f(x)\right\rangle \qquad (1)$$
- Key challenge: measurement collapses result to single basis state
  - To overcome this, quantum algorithms employ clever reduction schemes, making use of quantum interference effects

Outline (Beyond the Tangible?)

- Qubits, Quantum State
- Quantum Circuits
- *Q Algorithms*
- Q Compilers (Q Circuit Synth)
- Runtime Models
- Classical Microarchitecture

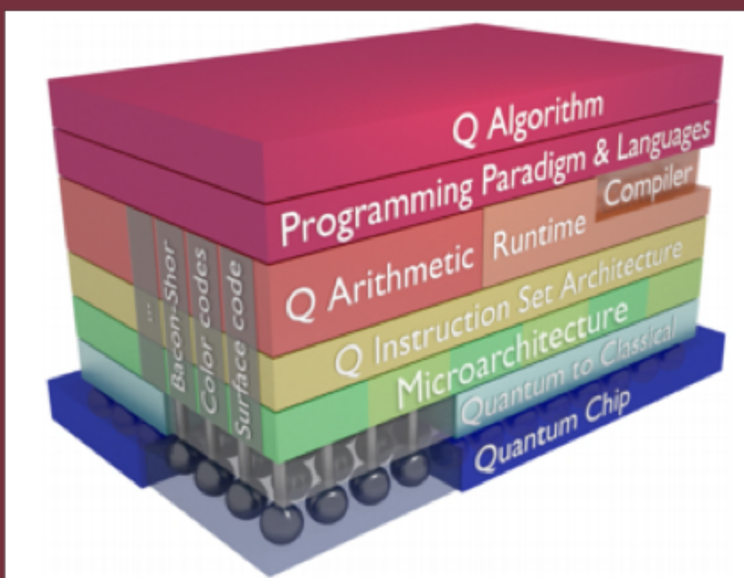
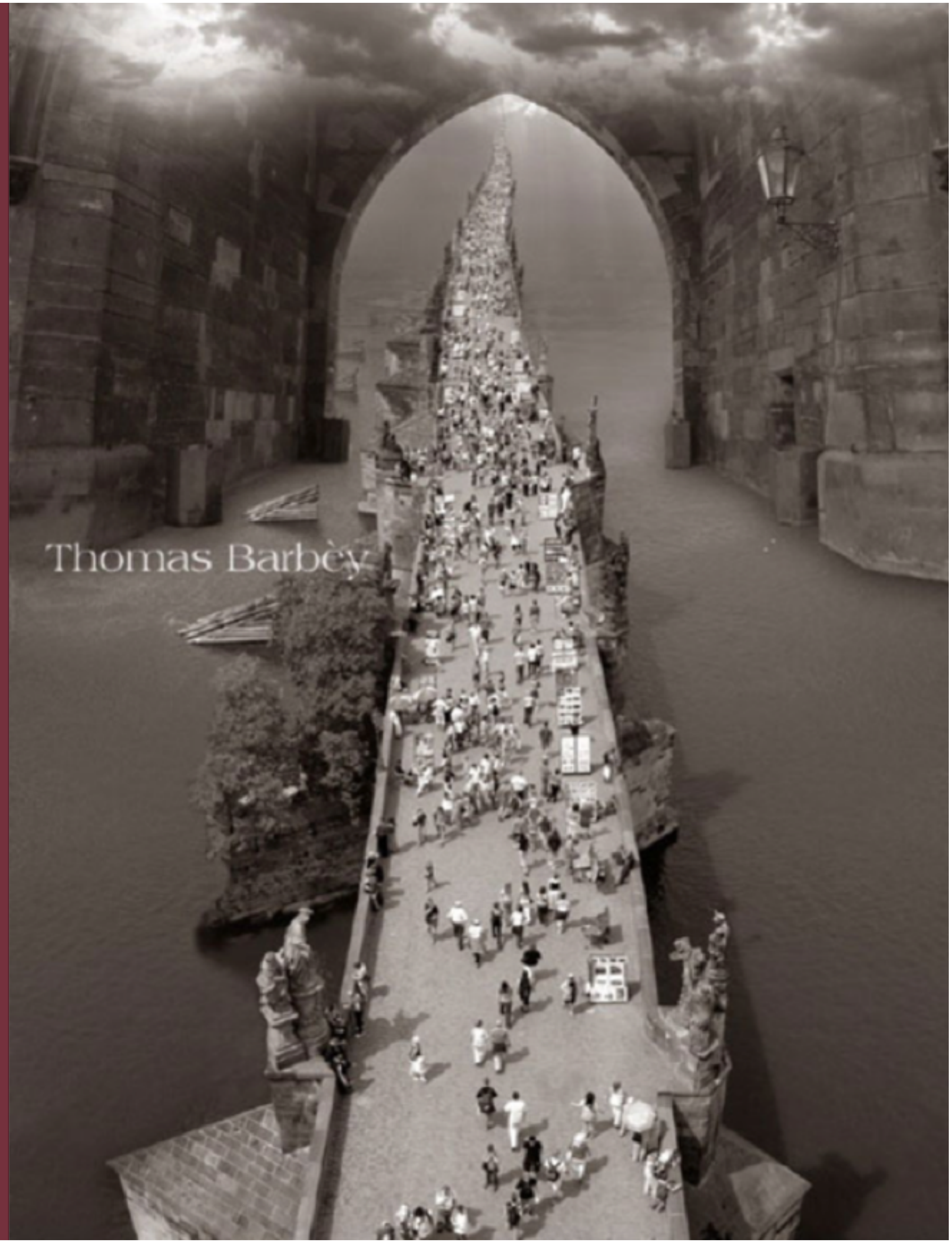
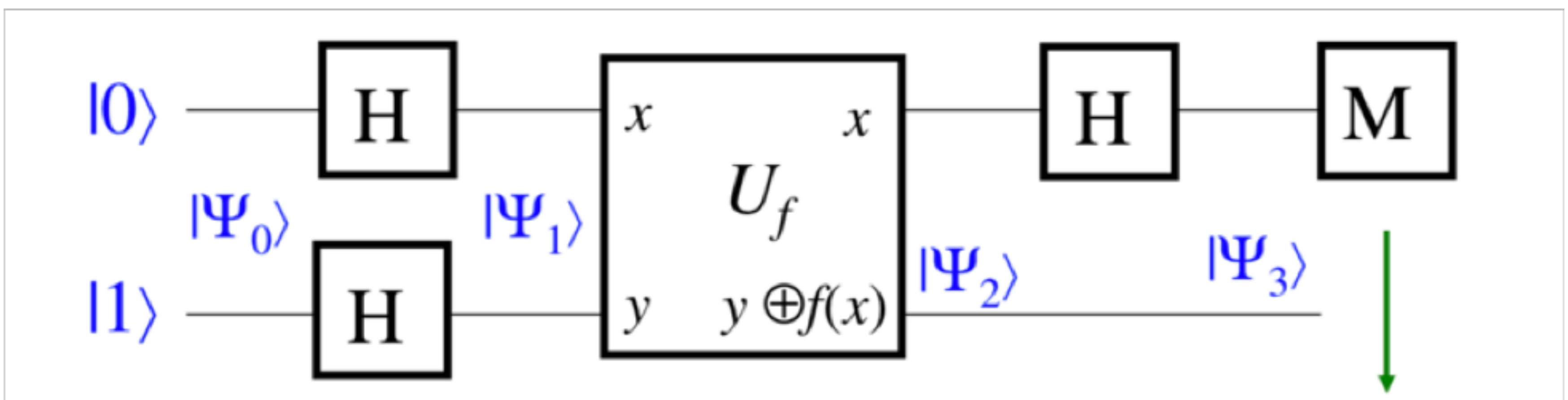


Figure from (Fu, 2017)



David Deutsch Algorithm and Circuit, 1985

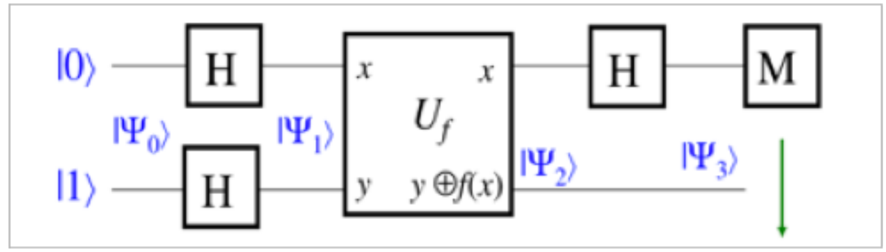
- Determine if a single-variable Boolean function f(x) is *constant* (f(0)=f(1)) or *balanced* (f(0) ≠ f(1)).
  - Classical version requires TWO runs of the algorithm
- Q Algorithm can evaluate f(0) and f(1) simultaneously
  - IDEA: quantum computer could extract the value of f(0)⊕f(1) at once (note this is 0 if f(x) constant and 1 if balanced)



Balanced = |1⟩
Constant = |0⟩

(Hayes, 2003)

Deutsch Algorithm Contd



Balanced = $|1\rangle$
Constant = $|0\rangle$

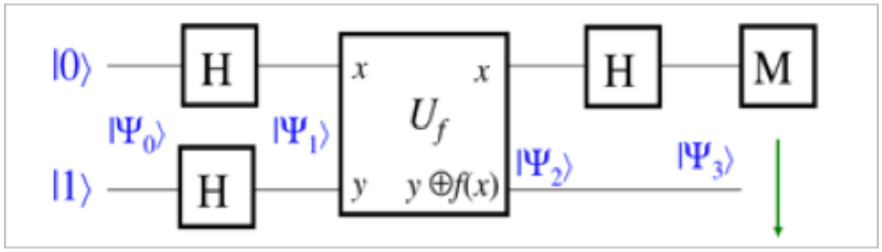
- Hadamards to create a *superposition of states*
  - After that a measurement would yield 50% likely one of the basis states

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}.$$

$$|0\rangle \text{ to } \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad |1\rangle \text{ to } \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

  - Creates joint state of $|\psi_1\rangle$ ($|y\rangle=H|1\rangle$, $|x\rangle=H|0\rangle$)
- We then utilize a custom unitary transformation U_f to compute f(x)
  - Input is top qubit, bottom output has y xor f(x)
- Hadamard is used again to interfere $|\psi_2\rangle$ states, yielding $|\psi_3\rangle$
- Measure top qubit (aka control qubit) for result

Deutsch Algorithm Contd



Balanced = |1⟩
Constant = |0⟩

- Recall what inputs |x⟩ and |y⟩ were after initial Hadamards

|y⟩

$$|0\rangle \text{ to } \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad |1\rangle \text{ to } \frac{|0\rangle - |1\rangle}{\sqrt{2}} \qquad (1)$$

$$U_f|0\rangle|y\rangle = \frac{U_f|0\rangle|0\rangle - U_f|0\rangle|1\rangle}{\sqrt{2}} = \frac{|0\rangle|f(0)\rangle - |0\rangle|f(0)\oplus 1\rangle}{\sqrt{2}} = (-1)^{f(0)}|0\rangle|y\rangle;$$

$$U_f|1\rangle|y\rangle = \frac{U_f|1\rangle|0\rangle - U_f|1\rangle|1\rangle}{\sqrt{2}} = \frac{|1\rangle|f(1)\rangle - |1\rangle|f(1)\oplus 1\rangle}{\sqrt{2}} = (-1)^{f(1)}|1\rangle|y\rangle. \qquad (2)$$

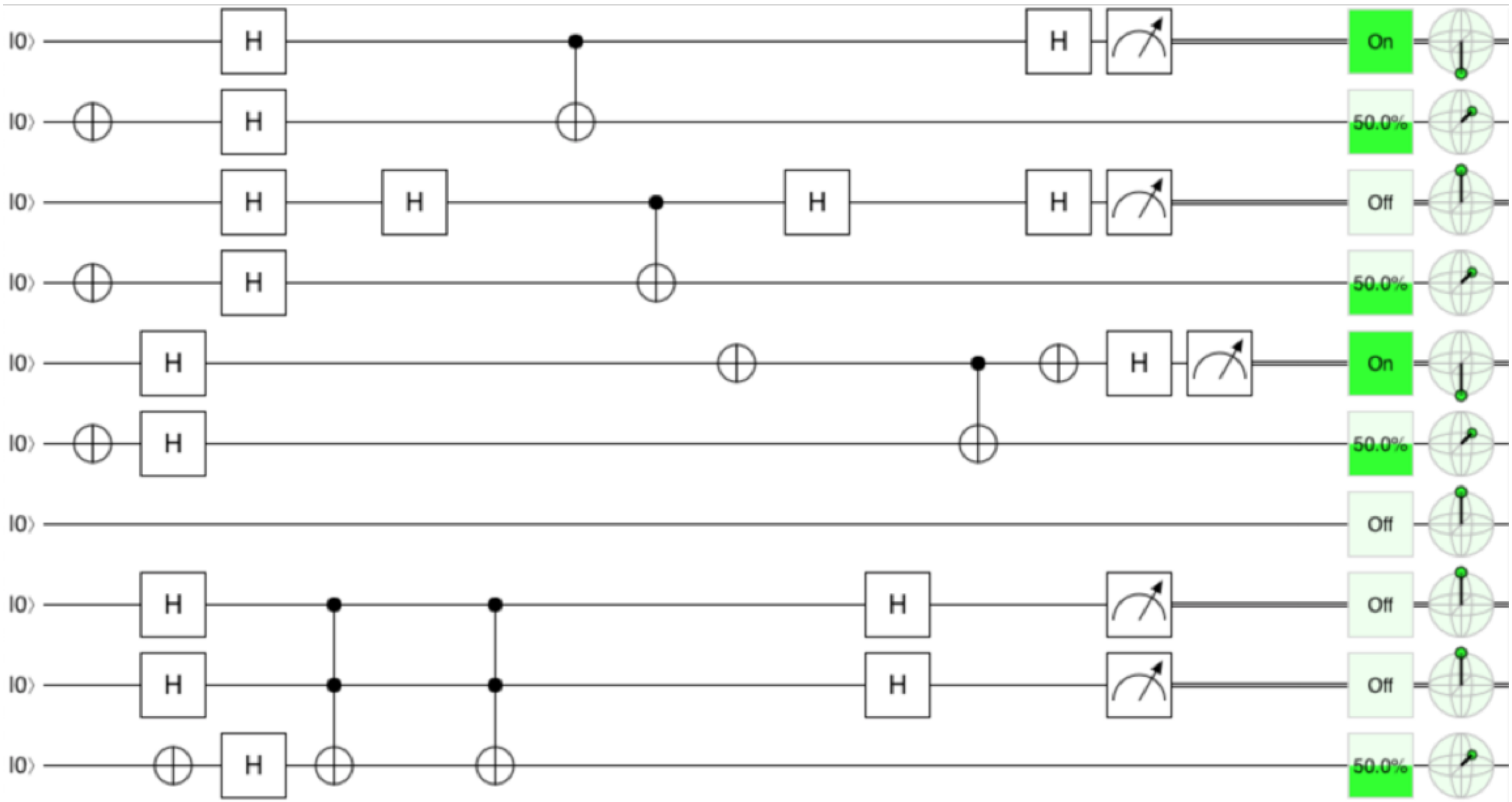
$$U_f|x\rangle|y\rangle = \frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}}|y\rangle = (-1)^{f(0)}\frac{|0\rangle + (-1)^{f(0)\oplus f(1)}|1\rangle}{\sqrt{2}}|y\rangle. \qquad (3)$$

$$(H\otimes I)U_f|x\rangle|y\rangle = \begin{cases} |0\rangle|y\rangle & , \quad \text{if } f(0) = f(1); \\ |1\rangle|y\rangle & , \quad \text{if } f(0) \neq f(1). \end{cases} \qquad (4)$$

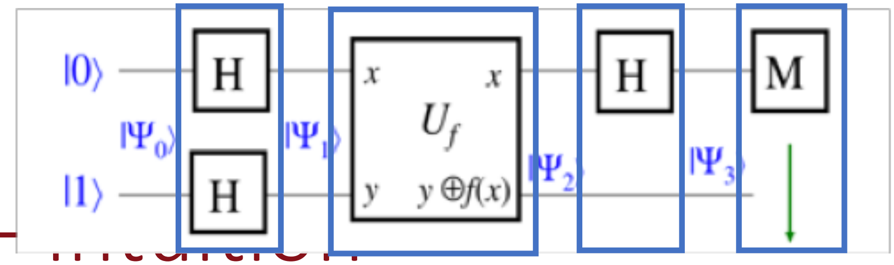
When measure to get result!

What we need to know

Deutsch Simulations



Deconstructing Deutsch –



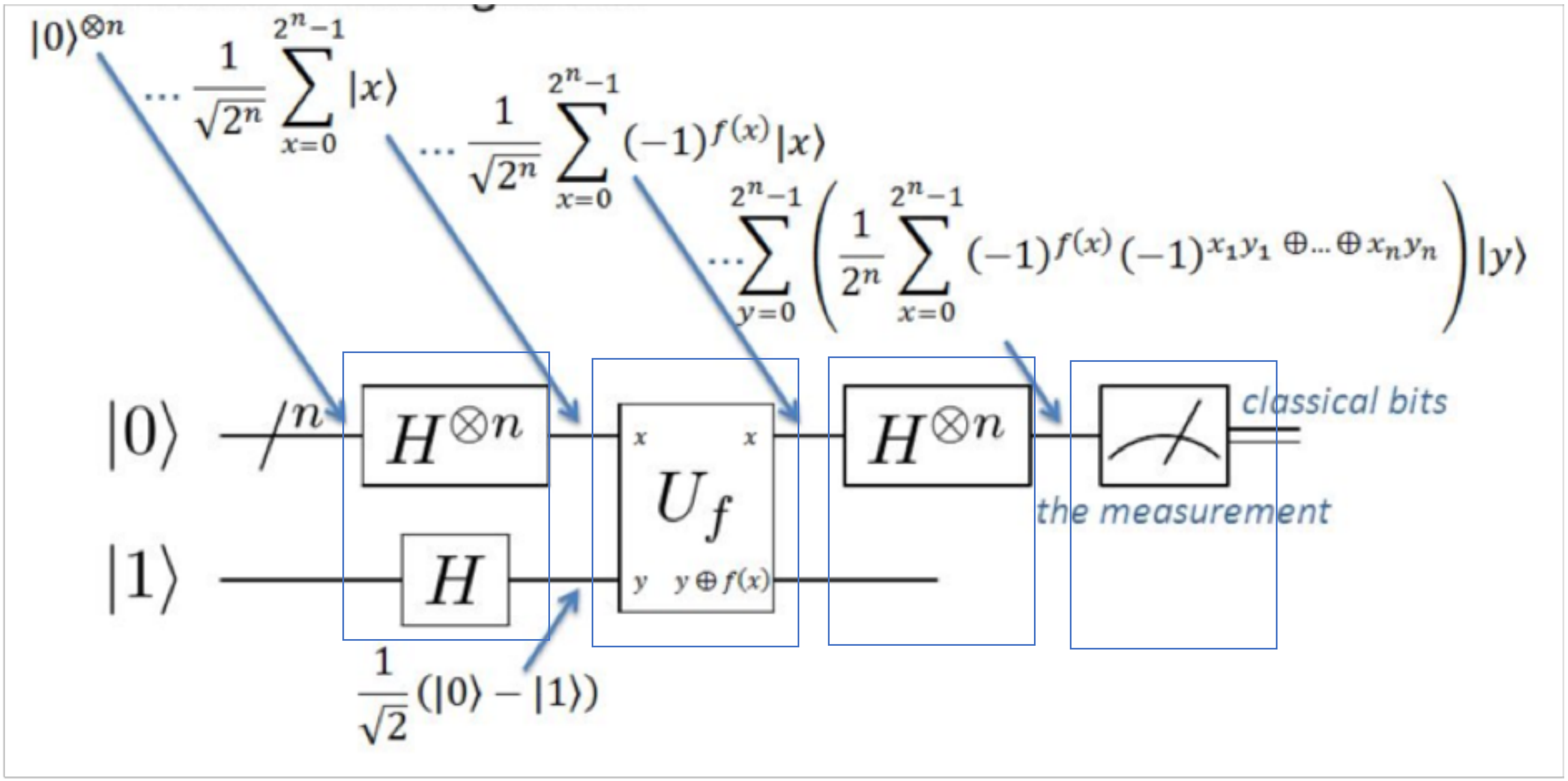
Balanced = |1⟩
Constant = |0⟩

- What did it take?

1. Algorithm/idea:
   - f(0) xor f(1) gives the result of whether f(x) is *balanced* or not
2. Cleverly using quantum computer:
   - Quantum parallelism – compute on superposition of basis states (prepared by/after the Hadamard gates) – SIMD like
     - Yielded result (U_f) had both f(0) and f(1) in it; in fact it had f(0) xor f(1)!
   - U_f is black box/oracle - f(x) but made reversible
   - Interfere/Bias - Why Hadamard at the end? Recognizing that |x〉 after U_f step is either H|1〉 or H|0〉 based on the result of f(0) xor f(1). Can map back to basis (preparing for a measurement) by H again: to |0⟩ if f is constant and |1⟩ balanced.

- Lesson: we need to think in terms of quantum parallelism & reduce result to global property combining simultaneous evaluations of f.

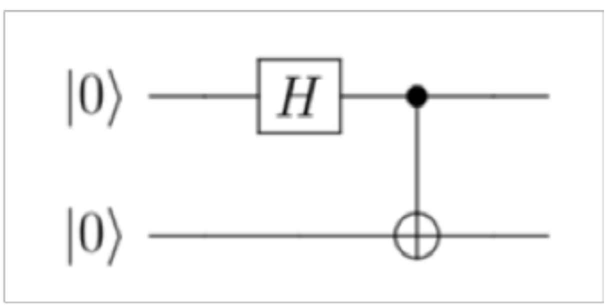
Deutsch-Jozsa (1996)



No Cloning, Power of Entanglement

- <u>No cloning theorem</u> – no ways to create a copy of $|\psi\rangle$ - this is a disadvantage vs classical computing
    - There is, however, a way to <u>assign a state to another qubit; needs entanglement</u> ... only one version can exist at the time

- <u>Entanglement</u> – state of qubits where they are correlated; one cannot express/decompose to tensor product of individual states (recall we used the tensor product for the joint state of two qubits)

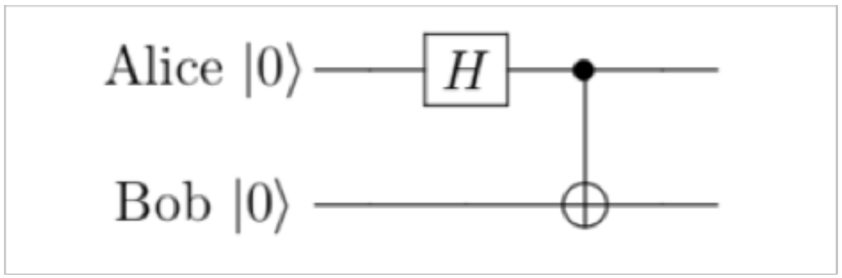


$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- EPR pair – after Einsten, Podolsky, and Rosen

Secret Sharing, State Assignment, Teleportation

- One qubit is Alice's and one Bob's. They entangle them as shown.

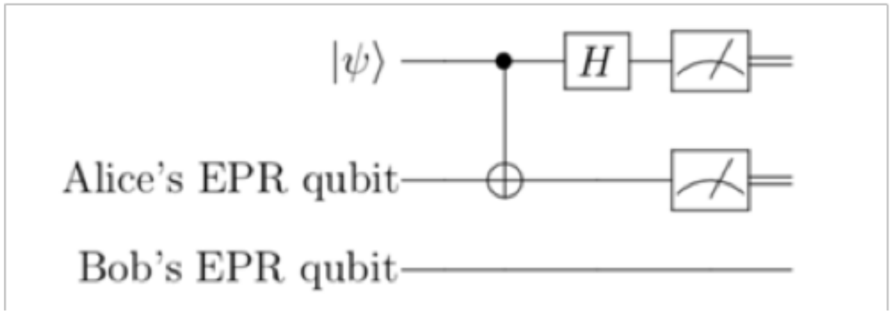


- They take them each home ☺
- Alice has another secret qubit $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$, wants to give it to Bob
  - Problem:
    - α_0 , α_1 cannot be extracted – measurement would destroy $|\psi\rangle$
    - Even if possible, at what precision? Complex numbers…many many bits.
  - Is it possible?

(O'Donnel, 2015)

Secret Sharing, State Assignment, Teleportation

- Alice can send |ψ⟩ with following circuit



- Let us see joint state before the CNOT

$$|\psi\rangle \otimes \left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right) = \frac{\alpha_0}{\sqrt{2}}|011\rangle + \frac{\alpha_0}{\sqrt{2}}|000\rangle + \frac{\alpha_1}{\sqrt{2}}|100\rangle + \frac{\alpha_1}{\sqrt{2}}|111\rangle. \quad (1)$$

- Then passes through the CNOT + H gates; joint three-qubit state:

$$\frac{\alpha_0}{2}|000\rangle + \frac{\alpha_1}{2}|001\rangle + \frac{\alpha_1}{2}|010\rangle + \frac{\alpha_0}{2}|011\rangle + \frac{\alpha_0}{2}|100\rangle - \frac{\alpha_1}{2}|101\rangle - \frac{\alpha_1}{2}|110\rangle + \frac{\alpha_0}{2}|111\rangle. \quad (2)$$

- Now ALICE measures her two qubits (top two in figure)

Secret Sharing, State Assignment, Teleportation

- Note that independent of ALICE's measurement Bob's state is equal or close to Alice's $|\psi\rangle$!

Alice's

Alice's measurement	Prob. of meas.	Collapsed state
$\|00\rangle$	$\frac{\|\alpha_0\|^2}{4} + \frac{\|\alpha_1\|^2}{4} = \frac{1}{4}$	$\|00\rangle \otimes (\alpha_0\|0\rangle + \alpha_1\|1\rangle)$
$\|01\rangle$	$\frac{\|\alpha_1\|^2}{4} + \frac{\|\alpha_0\|^2}{4} = \frac{1}{4}$	$\|01\rangle \otimes (\alpha_1\|0\rangle + \alpha_0\|1\rangle)$
$\|10\rangle$	$\frac{\|\alpha_0\|^2}{4} + \frac{\|-\alpha_1\|^2}{4} = \frac{1}{4}$	$\|10\rangle \otimes (\alpha_0\|0\rangle - \alpha_1\|1\rangle)$
$\|11\rangle$	$\frac{\|-\alpha_1\|^2}{4} + \frac{\|\alpha_0\|^2}{4} = \frac{1}{4}$	$\|11\rangle \otimes (-\alpha_1\|0\rangle + \alpha_0\|1\rangle)$

Bob's

- There is a unitary transformation (gate) for each case to $|\psi\rangle$
  - E.g., second row needs a NOT gate
  - third one a $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ fourth one a $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

- Note this is a **secret sharing approach. Alice needs to call the result of her measurement and Bob applies corresponding U**
  - First verified in 1992 by Bennett. In 2012, Ma, et. al., performed quantum teleportation at a distance of 143 kilometers.

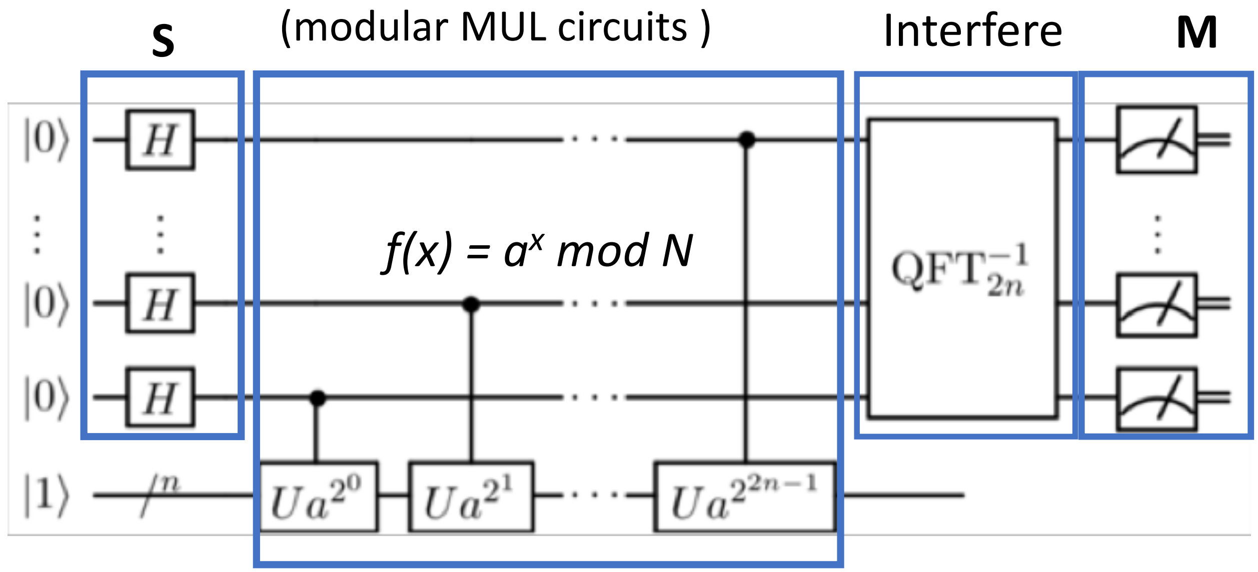
(O'Donnel, 2015)

Quantum Entanglement?

(Photography of Thomas Barbey)

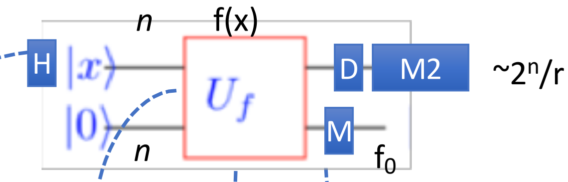
Shor's Algorithm 1994

Given an integer N,
find its prime factors



- Alg. Idea: calculate instead the *period r* of modular exponentiation.
- Math (simplified):
  - $f(x) = a^x \; mod \; N, a \; random$; period *r* smallest *int*. such as $f(x+r) = f(x)$
  - Classically compute *prime factors* as $\gcd\left(a^{\frac{r}{2}} + 1, N\right)$ and $\gcd(a^{\frac{r}{2}} - 1, N)$
  - How to calculate *r*?
- Quantum circuit **(see pattern S-Uf-I-M)** custom for each *N, a*
  - First part superposition, then next calculates f(x) with quantum parallelism
  - QFT gets you a number that is multiple of the inverse of r … (2^{2n}/r) => rest classically

Shor's Derivation (n top register assumed). Note I show for any periodic f(x)



$$|\Psi\rangle = H^{\otimes n}|0^n\rangle = \frac{1}{2^{n/2}}\sum_{x=0}^{2^n-1}|x\rangle. \quad (1)$$

$$U_f|\Psi\rangle|0^n\rangle = \frac{1}{2^{n/2}}\sum_{x=0}^{2^n-1}|x\rangle|f(x)\rangle. \quad (2)$$

$$\left(\frac{1}{\sqrt{m}}\sum_{k=0}^{m-1}|x_0+kr\rangle\right)|f_0\rangle.$$

(collapsed joint state)

$f(x_0) = f_0$ $\quad m = \lfloor \frac{2^n}{r} \rfloor.$ $\quad$ *r is period* (3)

(after measurement of f(x))

$$D\left(\frac{1}{\sqrt{m}}\sum_{k=0}^{m-1}|x_0+kr\rangle\right)$$
$$= \frac{1}{2^{n/2}}\sum_{y=0}^{2^n-1}\frac{1}{\sqrt{m}}\sum_{k=0}^{m-1}\omega^{(x_0+kr)y}|y\rangle$$
$$= \sum_{y=0}^{2^n-1}\omega^{x_0 y}\frac{1}{2^{n/2}\sqrt{m}}\left(\sum_{k=0}^{m-1}\omega^{kry}\right)|y\rangle. \quad (4)$$

$$\omega = e^{2\pi i/2^n}.$$

(after applying DQFT on top register)

$$\frac{1}{2^n m}\left|\sum_{k=0}^{m-1}\omega^{kry}\right|^2. \quad (5)$$

Maxed if $ry/2^n$ is an integer (6)

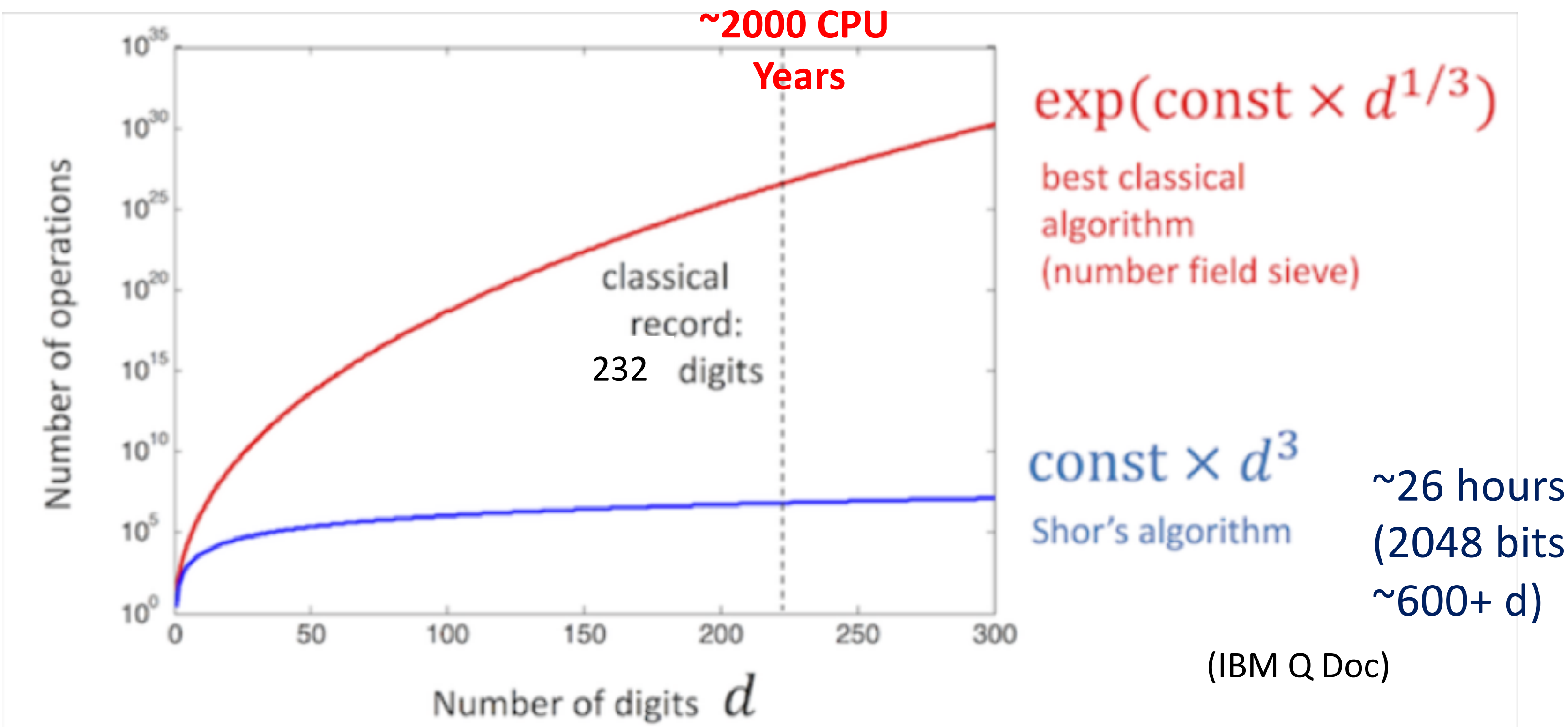
i.e., result will be multiple of $2^n/r$ (7)

(after measurement of top register)

Derivation follows (A. Dawar, Cambridge U)

Shor’s Algorithm Q Circuit

- Alexey Kitaev implementation: ~10d qubits (d is digits in N) and ~d^3 in gates



$$O\left(e^{1.9(\log N)^{1/3}(\log\log N)^{2/3}}\right) \rightarrow O\left((\log N)^2(\log\log N)(\log\log\log N)\right)$$

Impact on Today's Privacy, Security

- It will take around 26.7 hours for 2048 bits RSA (~600 digits) to be broken. Without fault tolerance it needs ~200M gates and ~6000 qubits.
- Also, a derivative of Shor's can be used to break ECC elliptic curve cryptography by computing discrete logarithms on a hypothetical quantum computer.
- The latest estimates for breaking a curve with a 256-bit modulus with 128-bit security level are 2330 qubits.
- Fault tolerance needs can significantly increase these numbers.

So, How Do You Design a Q Algorithm?

- Magic a la Ramanujan?
- Possible way to think (~ the 3Bs of Eagalman):

  - **Find** an f(x) as part of your problem, its **global property**, preprocess in classical

  1. **Setup superposition state(s) on input register**
  2. **Calculate simultaneously (SIMD) w/ quantum parallelism** f(x)
  3. **Bias/Interfere, Initial Measurement** - Bias state. Find way to expose metric across function outputs, a <u>global property</u> of f(x), to help solve what you need.
     - Measure f(x) … typical entanglement (f(x), x), f(x) property *reveals* itself in x.
  4. **Ready to Measure Result**
     - *The basis state that is most likely, must indicate your global property*

  - Finish classical

Design Requirements for a Q Computer

- The DiVincenzo Criteria for Q Hardware:
  - (i) scalable physical system with well-characterized qubits,
  - (ii) ability to initialize the state of the qubits,
  - (iii) "universal" set of quantum gates,
  - (iv) long relevant decoherence times, much longer than the gate-operation time,
  - (v) a qubit-specific measurement.

- Additionally, for overall system:
  - Q Compiler to generate from Q Program/Classical - sequence of gates w/ QEC
  - Classical CPU + Co-Processor w/ Controller to manage Q Hardware IO
  - Adequate fault tolerance - the *threshold theorem*:
    - Arbitrarily long Q computation with arbitrary reliability can be executed, if the error rates of Q gates are under a *critical accuracy threshold*. If decoherence only source, then robust computation requires decoherence 10^4 times longer than 1 Q gate delay

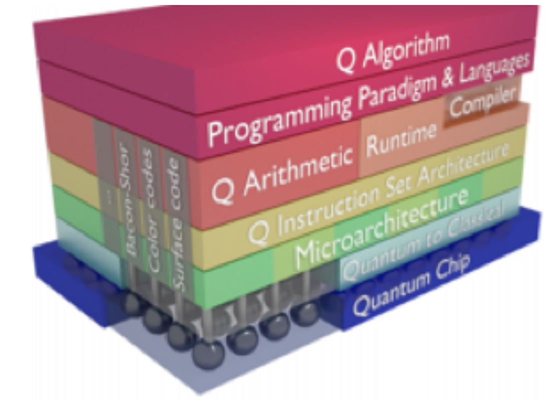
(E. Dennis, arxiv)

Why Use Q Tools/Compiler?

- Why not design algorithm and then create circuit manually?

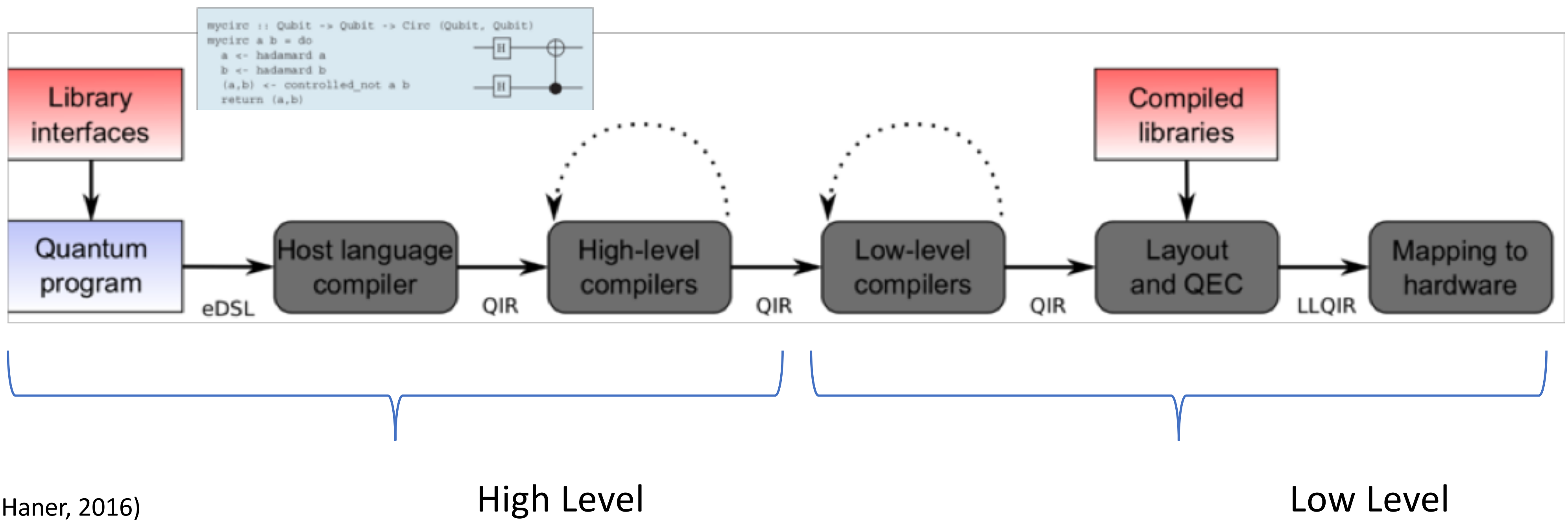
- Does not scale

- Beyond <u>convenience</u> (complexity), 3 primary problems to optimize
  1. Space – # of qubits needed
     - Gate sequences, Q oracles, QEC, manage ancillas/uncomputes; classical
  2. Time – decoherence, sequence of gates, precision
  3. Errors – how much error correction and where?
     - Maximize probability of correct interpretation



Q Compilers

- Classical code and embedded quantum program (EDSL)
- After the high-level compilation stage, the code consists of quantum gates, inlined library functions, and library calls to be resolved later
- Low- level compiler is to translate all quantum gates into sequences of gates from a discrete, technology-dependent set., & add QEC

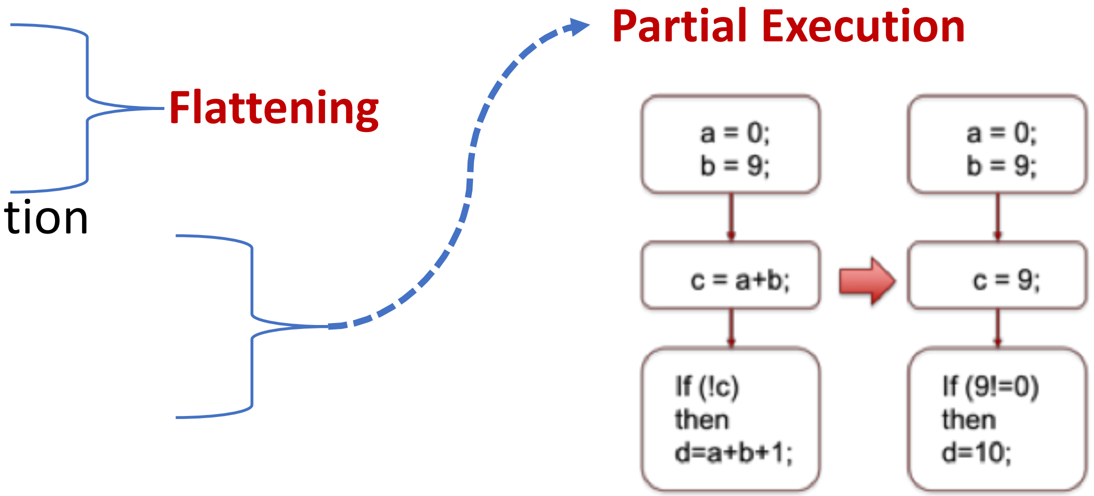


(Haner, 2016)

Prepare for Q Circuit Instantiation and Q Program Analysis

- <u>Prepare circuits</u>: have *inputs and oracle gates* <u>specified, +QEC</u>
  - Q Libraries can be ready but oracle needs to be generated (like in RTL)
  - May need to *generate multiple circuits at different sizes, inputs*

- Function inlining
- Loop unrolling
- Constant folding
- Constant propagation

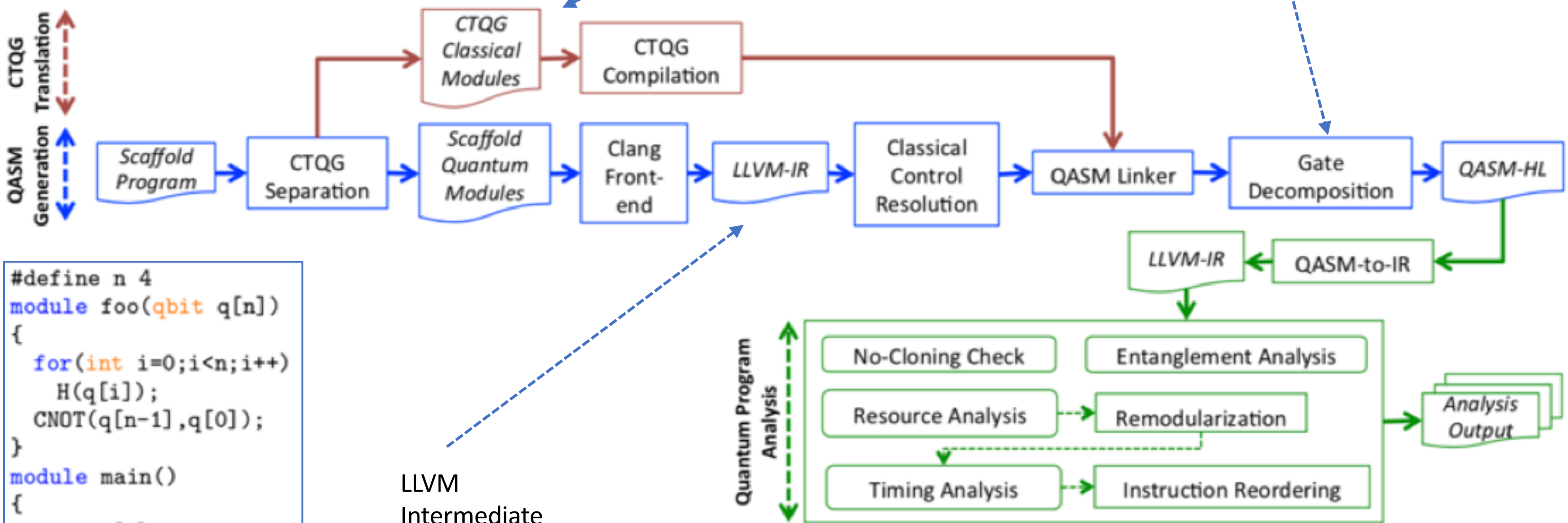


- <u>Quantum Program Analysis</u>: data-flow to check on entangled states, verify incorrect copying/assignment, resource utilization (incl ancillas), check uncomputes, estimate critical path

ScaffCC (C Front)

Allows using of classical gates

Toffoli, rotations replaced w/ subcircuits




```
#define n 4
module foo(qbit q[n])
{
  for(int i=0;i<n;i++)
    H(q[i]);
  CNOT(q[n-1],q[0]);
}
module main()
{
  qbit b[n];
  foo(b);
}
```

LLVM Intermediate Format

Q Program Analyses on LLVM

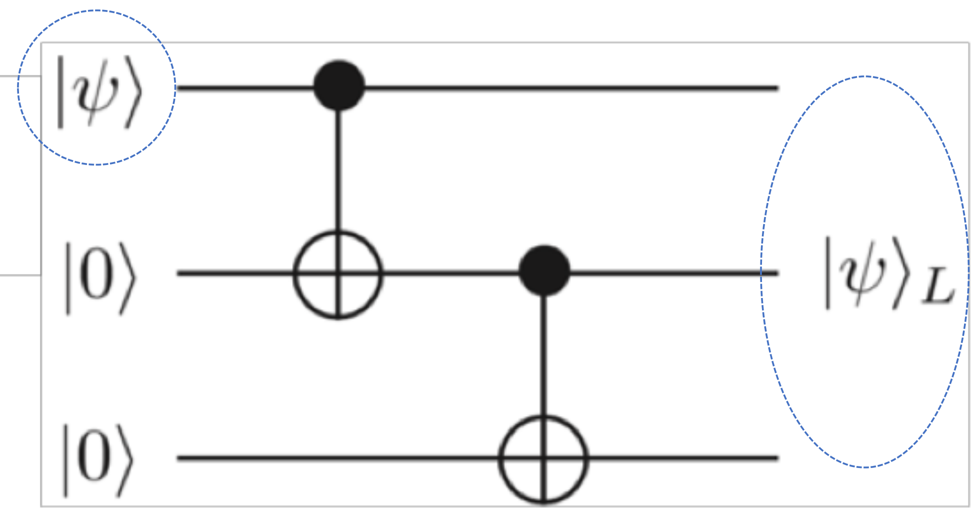
Scaffold code

(Abhary, 2014)

Low-level Optimizations, Error Handling

- Logical qubits to physical qubits (e.g. for QASM or hardware)
- Need to add redundancy/fault tolerance for state errors
  - Success – correct “interpretation” of results
  - Quantum Error Correction codes – bitflips and phase, QEC Logical qubits
    - Challenge – no state copies can be kept, hard to judge/detect due to measurement; need to use ancillas and *syndrome* information

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle_L + \beta|1\rangle_L = \alpha|000\rangle + \beta|111\rangle = |\psi\rangle_L .$$

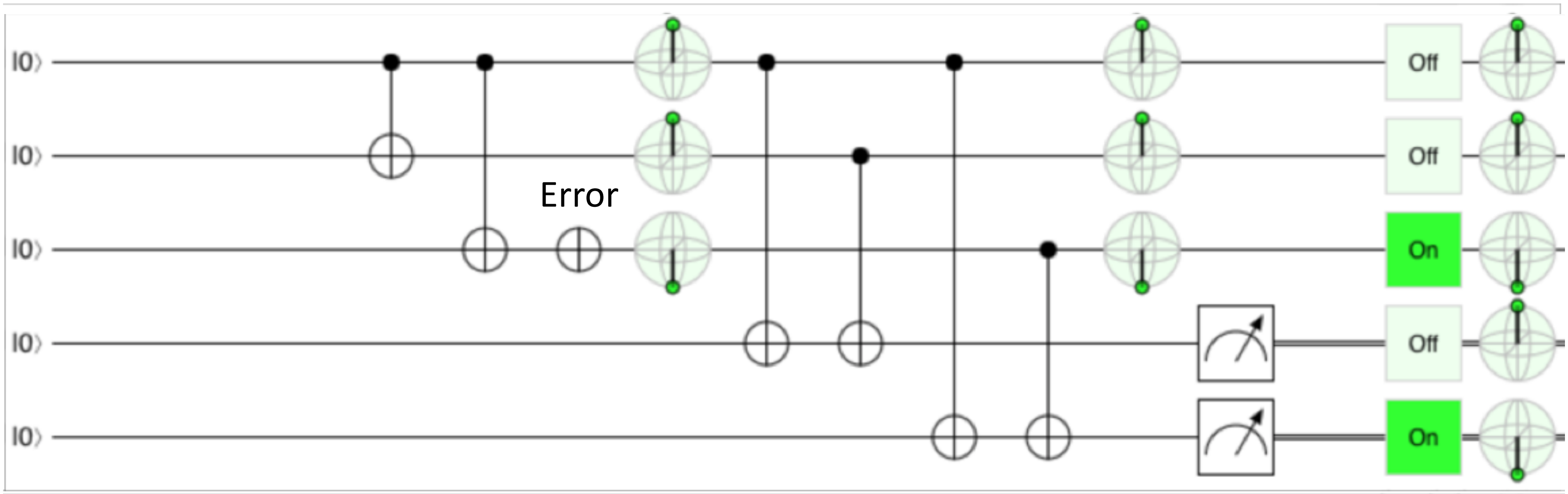


(Nemeto, 2013)

Part of 9-qubit code, Shor’s, 1995

EC-1-bit flip error

Fragment of Shor's Syndrome Based QEC- with Q Circuit Simulator



Ancilla Measurement: $\|00\rangle$,	∴ Clean State
Ancilla Measurement: $\|01\rangle$,	∴ Bit Flip on Qubit 3
Ancilla Measurement: $\|10\rangle$,	∴ Bit Flip on Qubit 2
Ancilla Measurement: $\|11\rangle$,	∴ Bit Flip on Qubit 1

Error Detection w/ Post-Select, Reset, Rerun

- Detect but not correct (00 state = no errors); no info. to correct

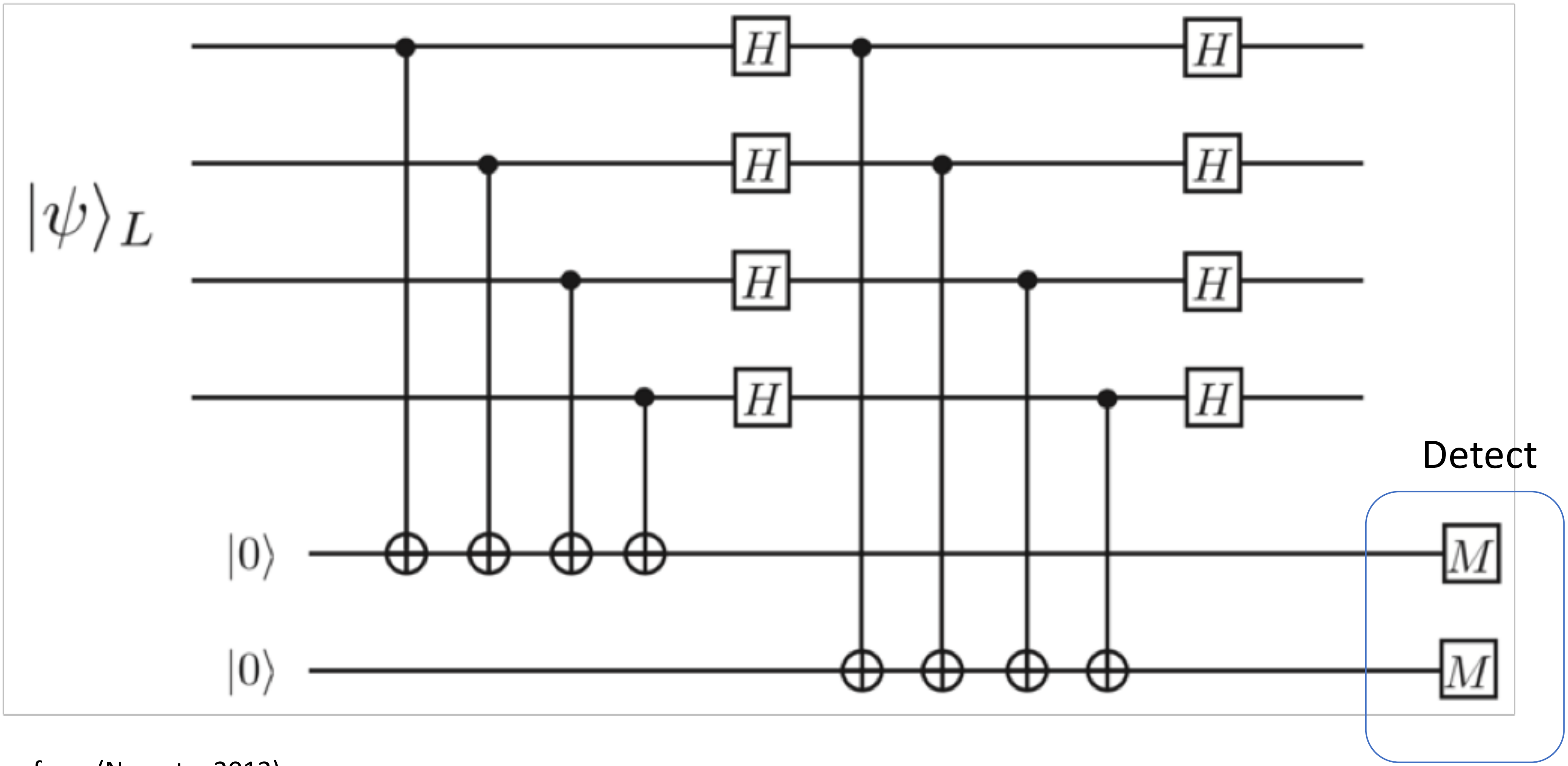


Figure from (Nemeto, 2013)

Stabilizer Formalism – Stabilizer Codes

- Enables easy synthesis of correction circuits/logical operations in Q compilers. E.g., 7 physical qubits for one logical. 6 Stabilizers are measured for single X (bit flip) or Z (phase flip) errors.

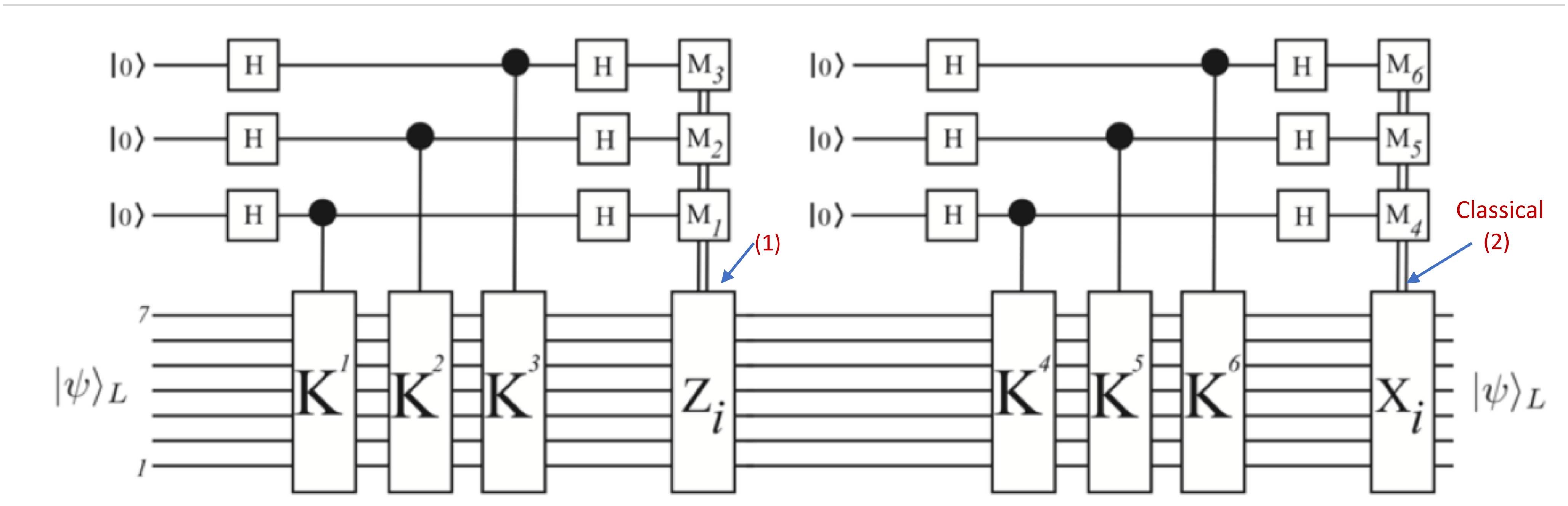


Figure from (Nemeto, 2013)

7-qubit *Steane* code, 1996

Envisioned Runtime Models

- Batch (Static)
  - Set up and run Q algorithm interleaved with regular codes statically
  - Trace based and w/ constant propagation – need circuit specified out
  - TBytes of code
- Dynamic Execution
  - Dynamically choose what to run - Interleave classical + Q arbitrarily
    - E.g., Post-selected error correction; which bit to add the X gate on out of 3
- *Dynamic Compilation*
  - Generate new circuit based on result of Q subroutine (measurement)

Is Dynamic Compilation a Must?

- Static version not always possible or best
- Phase estimation relying algorithms – phase needed to be extracted for next part of Q algorithm
  - Like in solving Linear Systems, Shortest Vector Algorithm
  - **<u>Angle of rotation depends on a measurement</u>**
- Note that even in Shor's; assumed that a static approach would work as phase rotations can be pre-generated (again code size! TBs of code, 1000s of high precision rotations)

$$\left\{R_z(\frac{\pi}{2^1}), R_z(\frac{\pi}{2^2}), R_z(\frac{\pi}{2^3}), ..., R_z(\frac{\pi}{2^k})\right\}$$

- Tight coupling of QHW-CHW in modern QEC like Surface codes
  - Adjust next phase of computation, compensate for errors detected

Code Size, Runtime Cost of Precision & QEC

- Ground State Estimation algorithm for Fe_2S_2 for example requires 10^{14} rotations, each approximated with 10^5 gates. That is 10^{19}!!!
- Dynamic compilation for a given phase precision based on static Solovay-Kitaev algorithm too slow
  - Initial work by Kudrow 5X improvement by classical optimizations
  - FP precision may not be adequate…

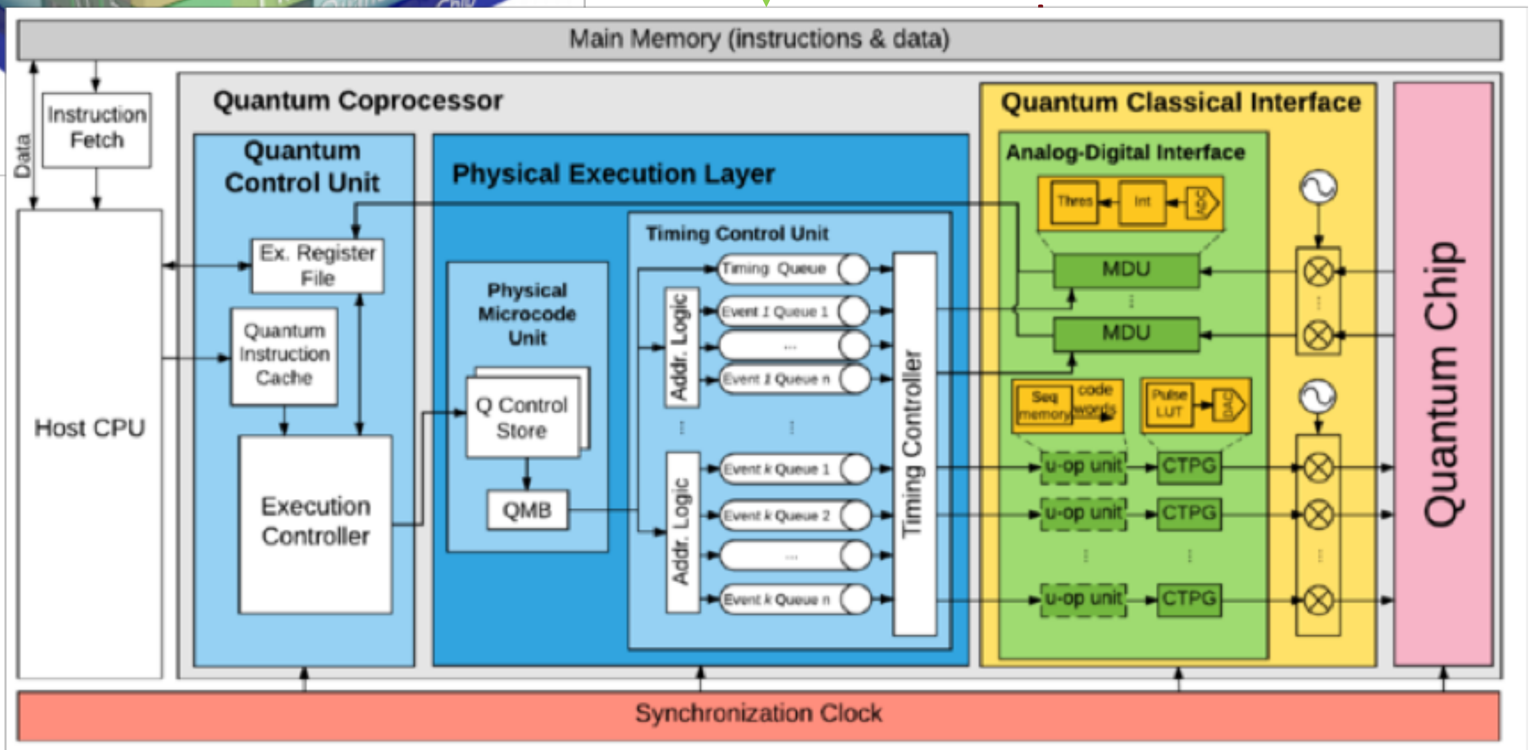
Technology	Experimental	Calculation
Ion Trap	10 μs [5]	1.0 μ s
Neutral Atom	31 μs [28]	0.915 μ s
Quantum Dot	2.6 ns [3]	1 ns
Superconductor	20 ns [23]	1 ns
Photons	15 ps [31]	1 ns

T Gate time; Bacon-Shor L2 QEC - 2 + orders slowdown

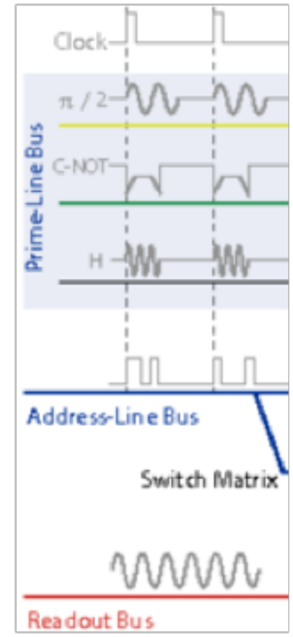
"… the high error-correction overhead of quantum computers can make the crossover point between polynomial and exponential performance occur at 100 years of computation"

(Kudrow et al, 2013 ISCA)

Microarchitecture Support – QuMA - Adds



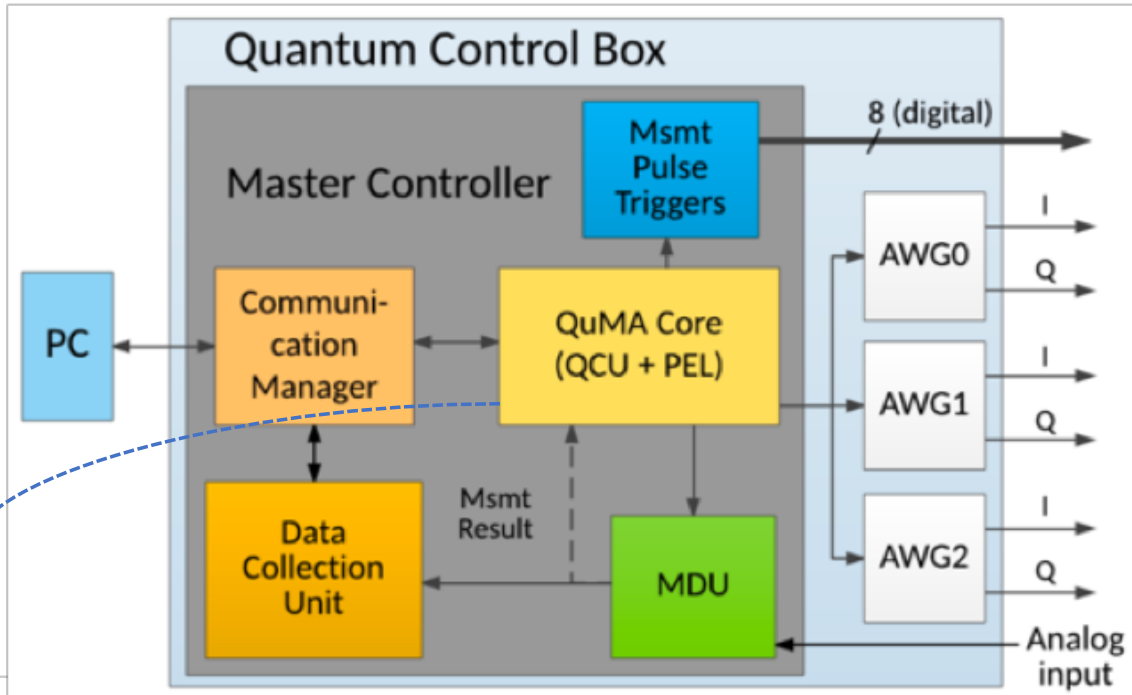
(Fu, 2017)



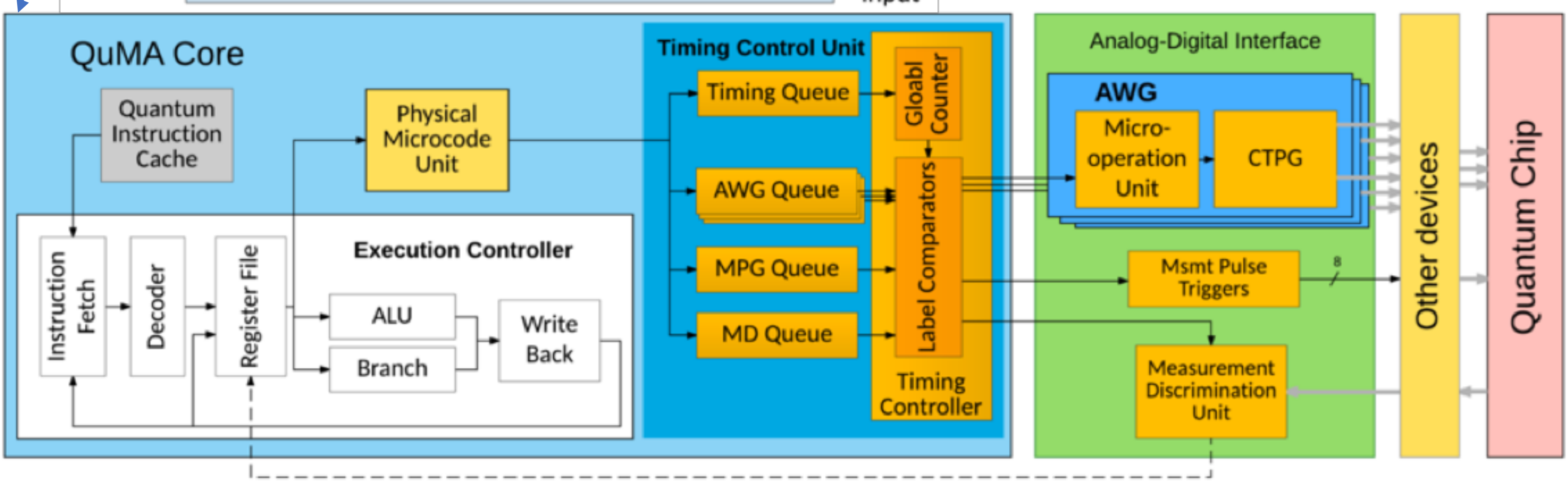
(Hornibrook, 2015)

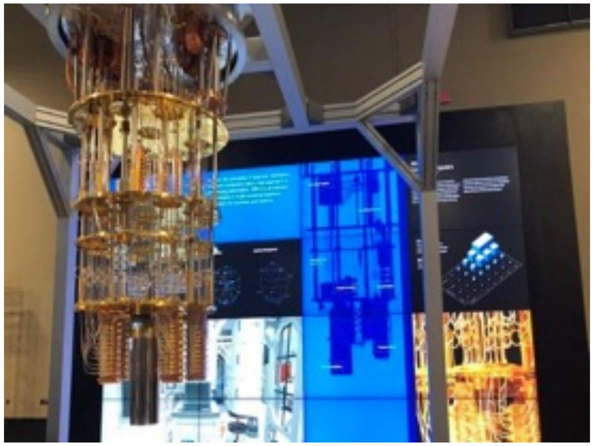
- Four concepts: (i) code-words (machine code), (ii) queue-based based timing; (iii) quantum instructions -> microcode -> code-words; (iv) QuMIS is a quantum microinstruction set. Validated on one super-cond Qb.
  - Manage events/timing, create analog control signals (pulses, wafeforms, coded in code-words)

Implementation w/ 4 FPGA Boards (Fu, 2017)



- Master w/ Altera Cyclone V 5CEFA9 FPGA chip, 8-bit ADC (for data from Q chip)
- 3 X boards - two-channel arbitrary waveform generator (AWG) each on Terasic DE0-Nano w/ Altera Cyclone IV EP4CE22F; 2X 14-bit DAC for qubit pulses
- 20 ns from codeword to ctrl





State-of-the-Art (Dec 2018)

- Simulator (best 56 qubits)
  - Microsoft's with Intel's AVX extensions. Plans "Brainwave" FPGA-based AI accelerator retargeted for Q. ATOS builds custom acceleration.
  - IBM 56-qubit general-purpose q system on a supercomputer. Harvard-MIT and the CALTECH simulated a 51-qubit quantum computer, but was not general-purpose. European researchers from Jülich Supercomputing Centre, Wuhan University, and the University of Groningen simulated a 46-qubits.
- Quantum Hardware (best 50 qubits at IBM, 72 at Google)
  - IBM 20-qubit chip in late 2017, internally tested a 50-qubit chip
  - Intel showed a 49-qubit chip at 2018 at the Consumer Electronics Show (CES).
  - Rigetti has19-qubit chip available for cloud access
  - Google Bristlecone 72 qubit in 2018 March
- Tool chain (widely available)
  - IBM Q Network QISKit API (1500 Univ, 35 papers), University tools based on LLVM (Scaffold, Haskell), NVIDIA QUDA, Microsoft Visual Studio IDE

Summary (+ Answer to Title Question)

Classical Computer	Quantum Computer (Future)
Bits (N bits “store” one of 2^N)	Qubits (Linear combination 2^N basis st)
Universal gate sets, Turing Machine, von Neumann, many-cores; Boolean	Quantum Universal gate sets, Quantum Turing Machine; Hilbert space
Classical inputs, outputs. I/O digital	Same nr of classical inputs, outputs. No loops, no copies. Reversible. I/O analog
Limited Data (SIMD), Instruction (ILP) and Task/Function Parallelism (TLP/FLP) Pipelining, caching.	(Almost) unlimited 2^n SIMD **Quantum Parallelism** (Tricks w/ entanglement, interference. Result: just global property)
Reliability: Perfect (almost)	High Error Rate. Surface QEC: 10^3-10^4 X overhead. 95%+ of work for errors. Skepticism on large QC **w/ 10M-1B Qb.**
Advanced Compilers	Initial flows – must deal w/ **code size explosion, dataflow likely hard**. Q Co-processors to manage *tight* dynamic.
Easy to write code, design algorithms	Can be mastered 😎! Do we need to find killer applications for small QC?

Summary on Security, Privacy

- RSA, ECC easily broken would large scale quantum computer materialize.
  - All email and messaging apps that rely on encryption alone.
  - Financial transactions, defense related communications.
  - All internet traffic.
- New methods are needed to encrypt or by utilizing ideas that do not fully rely on digital solutions alone
  - see EPRIVO physical separation approach.
  - Post quantum encryption algorithms.

Q U E S T I O N S

Thomas Barbey photography

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  - Or through EPRIVO service